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BY

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A TEXT-BOOK

ON

ROOFS AND BRIDGES

PART II
GRAPHIC STATICS

BY

MANSFIELD MERRIMAN

MEMBER OF AMERICAN SOCIETY OF CIVIL ENGINEERS

AND

HENRY S. JACOBY

PROFESSOR OF BRIDGE ENGINEERING IN CORNELL UNIVERSITY

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PREFACE TO THE FOURTH EDITION.

THE course of instruction in Roofs and Bridges presented in this Text-book consists of four parts. Part I deals with the computation of stresses in roof trusses and in all the common styles of bridge trusses; Part II treats of the determination of stresses by graphic methods. Part III presents the methods for the design of steel bridges, including proportioning of details and preparation of working drawings. Part IV discusses cantilever, suspension, movable, continuous, and arched bridges.

In the following pages the second part of this course is presented. The authors regard it as essential that students should completely work out a few typical cases like those here given in the six folding plates; also they consider it as important that students should solve many practical problems like those given at the ends of most of the articles. In this volume, as in Part I, the minimum as well as the maximum stresses are determined for each case; while all varieties of loading are treated, thus training students to use all kinds of specifications.

In Chapters I to VII alterations to the third edition have been made upon about thirty pages. In particular, Arts. 27 and 39 on loads for bridge trusses have been rewritten, and the new Art. 70 on a special construction has been added.

In this edition three chapters of new matter have been introduced. Chapter VIII, upon influence lines for stresses in simple bridge trusses, employs a new method which is perfectly general and may be applied as readily to a truss which is irregular in form or porportions as to any other. The method is graphic throughout and hence it is not necessary to employ the equations of influence lines as in the methods heretofore published in text-books. In modern practice the use of influence lines is restricted chiefly to trusses with inclined chords and with sub-divided panels, and hence only four types of trusses are used in the illustrative examples. This method of treatment is especially adapted to finding the loading and stresses in trusses of a new type like the *K* truss which has recently been introduced into this country. One article is especially devoted to that purpose.

It is believed that Chapter IX, on deflection influence lines, contains various improvements in the method of determining the deflection of beams. The graphic methods are completely illustrated by examples. Especial attention has been paid to the units of measure employed in making the diagrams so as to be in full accord with the fundamental principles of constructing equilibrium polygons.

Chapter X is devoted to classified references to numerous articles on graphic statics to be found in engineering periodicals and transactions. Many methods and hints of value to students and engineers are given in these articles. Nine full-page half-tone illustrations are also inserted in this Chapter.

Grateful acknowledgment for photographs are due to RALPH MODJESKI, JOHN E. GREINER, JOHN STERLING DEANS, ALBERT F. ROBINSON, and the late JOSEPH O. OSGOOD; for data and the use of a cut to the Shepard Electric Crane and Hoist Co.; also to CARL H. KNOETTGE for drawings from which the figures of Chapters VIII and IX have been made.

Compared with the last edition, the number of pages has been increased from 242 to 304 pages, and the number of figures from 138 to 162. The authors trust that this Part II is now better adapted than ever before to impart a thorough knowledge of graphic statics to students and to young engineers.

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GRAPHIC STATICS.

CHAPTER I.

PRINCIPLES AND METHODS.

ART. 1. THE FORCE TRIANGLE.

Statics is the science which deals with forces in equilibrium, and Graphic Statics is a method of solving statical problems by means of constructions on a drawing board. In the design of roofs and bridges numerous problems arise which may often be more conveniently solved by graphic constructions than by algebraic analysis.

A force is determined when its magnitude, direction, and line of action are known, and accordingly it may be graphically represented by the length, direction, and position of a straight line. Forces are usually given in pounds or kilograms, while the lengths of lines are measured in inches or centimeters. If the scale of force be taken as 2 000 pounds to one inch, then a line 1.43 inches long will represent a force of 2 860 pounds, and a force of 8 180 pounds will be represented by a line 4.09 inches long. A load is a force.

The resultant of two or more forces is a single force which produces the same effect as the forces themselves, and may therefore replace them. Let two forces P_1 and P_2 which act in the same plane upon the point *m* be represented in magnitude and position by the lines *mn* and *mp*, and in direction by the arrows. Let the parallelogram be completed by drawing a line through *n* parallel to P_2, and

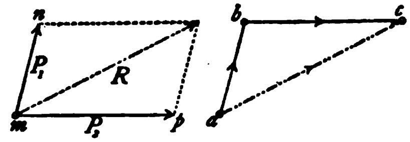


Fig. 1.

a line through p parallel to P_1, and then let m be joined with their point of intersection. This line, designated by R, represents the resultant of the two given forces. To find the magnitude of this resultant by the analytic method, let θ be the angle included between P_1 and P_2; then from either of the triangles composing the figure a well known theorem of geometry gives

$$R^2 = P_1^2 + P_2^2 + 2P_1P_2 \cos \theta.$$

For instance, if $P_1 = 215$ pounds, $P_2 = 514$ pounds, and $\theta =$ 75 degrees, there is found by computation $R = 606$ pounds.

The graphic method of finding the resultant consists of the following operations: On a sheet of paper, with the help of a ruler and protractor, from a point m two indefinite lines are drawn, making an angle of 75 degrees with each other. Using a suitable scale, say of 100 pounds to the inch, the distance mn is made 2.15 inches for the value of P_1 and mp is made 5.14 inches for the value of P_2. The parallelogram is then completed with ruler, triangle, and pencil, and the line mn is found by the scale to be 6.06 inches long; hence $R = 606$ pounds.

It will be seen that it is not necessary to construct the entire parallelogram, since the triangles on the opposite sides of the diagonal are equal. The triangle above the diagonal can be constructed by drawing a line through n parallel to P_2, laying off upon it the value of P_2, and then joining its end to m; similarly the lower triangle can be independently drawn. Either of these triangles is called the force triangle.

Usually the lines of action of the given forces form part of a diagram upon which it is not desirable to construct the force triangle. In this case let any suitable point a be selected, and ab be drawn parallel and equal to P_1; then through b let bc be drawn parallel and equal to P_2, and let a be joined with *c*. The line *ac* represents the magnitude of the resultant R, and is measured by the same scale as that used in laying off *ab* and *bc*. The direction in which the resultant acts is indicated by the arrow upon *ac*, and this is seen to be opposed to the directions of those upon *ab* and *bc* in following around the triangle. Finally, the line of action of the resultant R must pass through *m*, the point of application of the given forces P_1 and P_2. Hence the resultant R is found in magnitude, direction, and line of action by drawing through *m* a line equal and parallel to *ac*.

The above operation is termed composition of forces, P_1 and P_2 having been combined into one. The reverse process of resolution of forces may also be effected by the force triangle. For instance, let R_1 in Fig. 1 be given, and let it be required to find its components in the directions of *mn* and *mp*. Let *ac* be drawn equal and parallel to R, and through its extremities let *ab* and *cb* be drawn parallel to the given directions; these lines intersect in *b*, and when they are measured by the scale the magnitude of the components P_1 and P_2 will be known. Lastly, through *m*, the point of application of R, let P_1 and P_2 be laid off in the given directions, equal to *ab* and *bc*, and the lines of action of the components are determined.

Several forces are said to be in equilibrium when no tendency to motion is produced in the body upon which they act. In Fig. 1 suppose a force, P_3, equal and opposite to R, to be applied at *m*; then this force together with P_1 and P_2 will be in equilibrium, for the last two may be replaced by their resultant R, which by the conditions specified holds P_3 in equilibrium. The corresponding force triangle will be *abc* with the direction of *ac* reversed, so that all the forces around the triangle have the same direction; hence, when three forces are in equilibrium, they form a closed force triangle.

When three forces whose lines of action lie in a plane and intersect in one point are in equilibrium, any one may be determined when two are given. In Fig. 2 let P_1 and P_2 be given to find P_3. Let *ab* be laid off equal and parallel to P_1, and from *b* let *bc* be drawn equal and parallel to P_2: then *ca*, the closing side of the triangle, represents P_3 in magnitude and direction. As its line of action must also pass through *m*, the force P_3 is drawn equal and parallel to *ca*, and in the same direction, thus completing the solution.

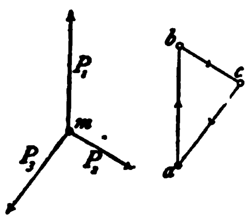


Fig. 2.

Should only one force be given, together with the lines of action of the other two, their magnitudes and directions may be found. Let P_1 and the lines of action of P_2 and P_3 be given. Draw *ab* parallel to P_1, mark off its length according to scale, and through its extremities draw lines parallel to P_2 and P_3; these lines intersect in *c*, and the length of *bc* gives the magnitude of P_2, its direction being from *b* to *c*, while *ca* represents P_3 in the same respects.

The force triangle is the foundation of the science of graphic statics. By it all problems relating to the composition and resolution of forces can be solved, when the forces are but three in number and act in the same plane upon a common point.

Problem 1. Find the magnitude of the resultant of two forces making an angle of 60 degrees with each other, one being 25 pounds and the other 40 pounds.

Prob. 2. The lines of action of two forces, of 50 and 30 pounds respectively, make an angle of 120 degrees. What is the magnitude of the force that holds them in equilibrium and the angles that it makes with each of them?

ART. 2. THE FORCE POLYGON.

When it is required to find the resultant of a number of forces acting in the same plane and having a common point of application, the resultant of two of the forces may be found by Art. 1, a third force may then be united with it to obtain a second resultant, and this operation continued until all the forces are combined. In Fig. 3, the line R_1 is the resultant of P_1 and P_2, the line R_2 is the resultant of R_1 and P_3, and R is the resultant of R_2 and P_4, and therefore of the given forces P_1, P_2, P_3, and P_4. It is, however, not necessary to construct these resultants in order to find R, if the dotted lines be drawn parallel and equal to P_2, P_3, and P_4.

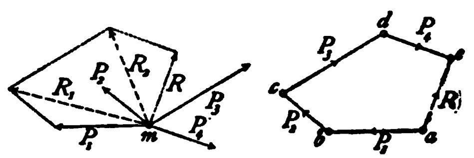


Fig. 3.

The polygon shown by broken lines is called the force polygon; the resultant R forms its closing side, and each of the other sides represents one of the given forces. The diagram *abcde* shows the polygon as it is generally drawn with the diagonals omitted. The direction of the resultant is opposed to the direction of all the given forces in following around the sides of the polygon; thus the arrow on *ae* has the reverse direction of the other arrows.

The force polygon may therefore be constructed as follows:

> Draw in succession lines parallel and equal to the given forces, each line beginning where the preceding one ends, and extending in the same direction as the force it represents. The line joining the initial to the final point represents the resultant in direction and magnitude.

To produce equilibrium with P_1, P_2, P_3, and P_4, a force equal and opposite to R must be applied at *m*. This added force in the force polygon is equal to *ea* with its former direction reversed, and the distance from the initial to the final point in the construction of the polygon becomes zero.

Hence, if a number of forces lying in the same plane and having a common point of application are in equilibrium, they will form a closed force polygon, and in passing around it all the forces will have the same direction.

In either of the above cases it makes no difference in what order the forces are arranged in the force polygon. Thus in Fig. 3 the sides of the force polygon are drawn in the order P_1, P_2, P_3, P_4, R; but the same value of R, both in intensity and direction, will be obtained if they are drawn in any other order, as, for example, P_3, P_1, P_4, P_2, R. Again, in Fig. 4, let the four

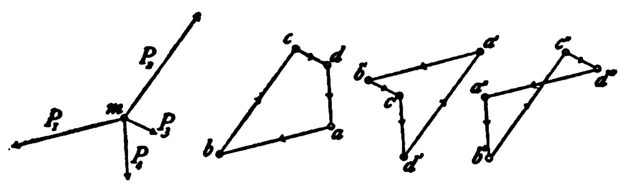


Fig. 4.

forces which meet at m be in equilibrium; then taking them in the order P_1, P_2, P_3, P_4 the force polygon $abcd$ is drawn, in the order P_1, P_3, P_4, P_2 the polygon $a'b'c'd'$ results, and in the order P_4, P_2, P_3, P_1 the polygon $a''b''c''d''$ is found, each of which graphically represents the given forces. In the last case it is seen that two of the lines in the force polygon cross each other; this is of frequent occurrence in practical problems.

The force triangle (Art. 1) is but a particular case of the force polygon, namely, when the forces are but three in number. The word polygon is hence often used in a general sense as including that of the triangle. From three forces in equilibrium two force triangles may be drawn; from four forces in equilibrium six force polygons can be formed.

Prob. 3. Draw a force polygon for five forces in equilibrium, and prove that any diagonal of the polygon is the resultant of the forces on one side and holds in equilibrium those on the other.

Prob. 4. Let $P_1 = 100$ pounds, $P_2 = 175$ pounds, and $P_3 = 60$ pounds, and let the angles which they make with each other be $P_1mP_2 = 135°$, $P_2mP_3 = 87°$, $P_3mP_1 = 138°$. Draw three force polygons and determine from each the value of the resultant, and the angle that it makes with P_1.

ART. 3. CONDITIONS OF EQUILIBRIUM.

When several forces lie in the same plane the necessary and sufficient conditions of static equilibrium are that there shall be no tendency to motion, either of translation or rotation. Analytically this is expressed by saying that the algebraic sum of the components, both horizontal and vertical, of the forces must be zero, and that the algebraic sum of the moments of the forces must also be zero.

When the given forces have a common point of application, the graphic condition for equilibrium is that the force polygon must close. For, if it does not close the line joining the initial with the final point represents the resultant of the given forces (Art. 2), and this resultant will cause motion; and if it does close there exists no resultant. Therefore, if the given forces which meet at a common point are in equilibrium the force polygon must close; and conversely, if the force polygon closes the given forces must be in equilibrium.

When several forces lying in the same plane have different points of application, so that their lines of action do not intersect in the same point, and are in equilibrium the force polygon must also close, since no resultant exists. Thus, suppose the given forces to be four in number, let the directions of two of these be produced until they intersect and their resultant found; this resultant must pass through the point of intersection of the remaining two forces, since equilibrium obtains. Hence the rule above established for forces acting at a common point applies also to this case, and the force polygon must close if they are in equilibrium.

If several forces have different points of application, and the force polygon does not close, the line joining the initial and final points represents the intensity and direction of the resultant. For, as a force can be considered as acting at any point in the line of its direction two of them may be combined into a resultant, and this resultant may be combined with one of the other forces, and so on until the final resultant is obtained in the same manner as in Art. 2.

If several forces have different points of application, and the force polygon closes, it is not necessarily true that the given forces are in equilibrium. For example, let a beam or stick be acted upon by three forces as shown in Fig. 5, the forces P_1, P_2, and P_3 being equal, P_1 and P_2 making an angle of 30 degrees with the horizontal and P_3 being vertical. It is plain that equilibrium is here impossible, and yet the force polygon *abc* closes. Upon reflection it will be seen that the equilibrium of the beam under the action of the three given forces can only be maintained by a couple, that is, by two equal parallel forces acting in opposite directions. It is because the resultant of the forces of a couple is zero that the force polygon closes in this case; and it will be found that in all instances of non-equilibrium where the force polygon closes that a couple is necessary to maintain equilibrium.

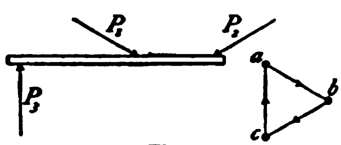


Fig. 5.

The above conditions apply to forces lying in one plane. It is rare in problems relating to roofs and bridges that forces acting in different planes need to be considered, and hence in the following pages it will always be understood, unless otherwise stated, that the forces under discussion lie in the same plane.

Prob. 5. In Fig. 5, let each of the forces be 100 pounds, and let the distance between the points of application of P_1 and P_2 be 4 feet, and between those of P_1 and P_3 be 5 feet. Compute the magnitude of the forces of a horizontal couple to maintain equilibrium when the vertical distance between their points of application is 3 inches. Draw the forces of the couple in both diagrams of Fig. 5.

ART. 4. STRESSES IN A CRANE TRUSS.

As an example of the application of the preceding principles let it be required to graphically determine the stresses in the members B_2, B_3, etc., of the crane truss shown in the left-hand diagram of Fig. 6, due to a load P_1 acting at the peak. The member B_2 is called the tie, B_3 the jib, B_4 the post, and B_5 the back-stay. The post is vertical and its length is 16 feet, the length of the jib is 30 feet, of the tie 21.5 feet, and of the back-stay 20 feet; from these dimensions the diagram of the crane truss is constructed. The load P_1 is 5 000 pounds.

Using a scale of 2 000 pounds to an inch, the construction of the stress diagram is begun by laying off *ab* parallel to P_1 and equal to $2\frac{1}{2}$ inches. Now at the peak the force P_1 is resolved into two forces whose lines of action are in the two members B_2 and B_3; hence, by Art. 1, draw *ac* parallel to B_2 and *bc* parallel to B_3, thus obtaining the force triangle *abc*; the length of *ac* gives the stress in B_2, and that of *bc* gives the stress in B_3. Next passing to the apex *n* the stress

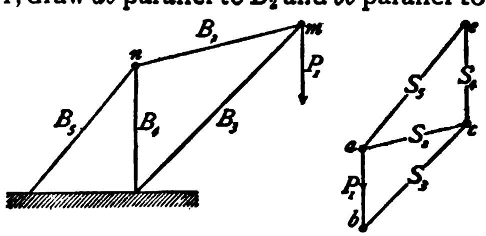


Fig. 6.

in B_2 is known and those in B_4 and B_5 are to be found; hence from *a* and *c* draw parallels to these members and these lines, intersecting at *e*, give *ce* as the stress in B_4, and *ae* as that in B_5. This completes the force diagram.

The next step is to determine the character of the stresses, that is, whether they are tension or compression. Beginning with the triangle *abc*, which represents the forces acting at the apex *m*, the direction of *ab* is known to be downward, hence following around the triangle (Art. 1) the stress S_3 acts from *b* toward *c*, and S_2 from *c* toward *a*; transferring these directions to the lines of action at the apex *m* it is seen that S_3 acts toward *m*, and is therefore compression, while S_2 acts away from *m*, and is therefore tension. Passing now to the apex *n* the stress S_2 in B_2 is known to be tension and hence it acts away from *n*, accordingly in the force triangle *cae* it acts from *a* toward *c*; hence S_4 acts from *c* to *e*, and S_5 from *e* to *a*; transferring these directions to the lines of action at *n* it is found that S_5 is tension and S_4 is compression.

Applying the scale to the lines of the force diagram the following results in pounds are now found, the sign + denoting tension and – denoting compression:

$$S_2 = +6\,700, \quad S_3 = -9\,350, \quad S_4 = -6\,850, \quad S_5 = +10\,800.$$

These are the stresses in the members B_2, B_3, B_4, and B_5 due to the load of 5 000 pounds acting at the peak. If 10 000 pounds were hung at the peak it is plain that each line of the force triangle would be twice as long as before, or the stresses in the members would be double the values above given.

The two parts of Fig. 6 are called the 'truss diagram' and the 'stress diagram' respectively. Each triangle in the stress diagram *abce* corresponds to the forces acting at one of the apexes in the truss diagram, so that it may be said that the two figures are reciprocal.

Prob. 6. In Fig. 6 let B_4 be vertical and let $B_2 = 30$, $B_3 = 45$, $B_4 = 50$, and $B_5 = 90$ feet. Draw the stress diagram and determine the stresses in all the members due to a force of 8 000 pounds which acts at an angle of 30 degrees to the right of the vertical drawn through the peak *m*.

ART. 5. STRESSES IN A POLYGONAL FRAME.

In Fig. 7 let $B_1B_2B_3B_4$ be a polygonal frame which supports the loads P_3 and P_4 and which is itself suspended by the forces P_1 and P_2 acting in two ropes. The frame being in equilibrium under the action of the exterior forces P_1, P_2, P_3, and P_4, it is required to find the stresses in the members B_1, B_2, B_3, and B_4.

As the exterior forces P_1, P_2, P_3, and P_4 are in equilibrium, the force polygon representing them must close (Art. 3). First, then, let the force polygon *abcd* be drawn, *ab* representing P_1

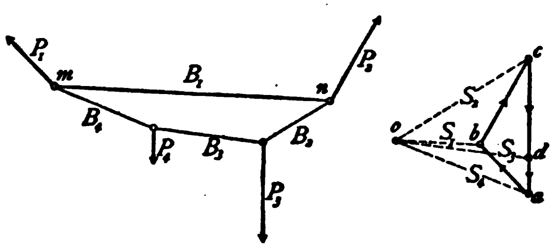


Fig. 7.

in magnitude and direction, *bc* representing P_2, and so on. Now at each apex of the polygonal frame there are three forces which are in equilibrium. Thus at *m* the force P_1 is known, and if from *b* and *a* lines be drawn parallel to B_1 and B_4 these intersect in *o*, giving *bo* as the stress S_1 in B_1 and *ao* as the stress S_4 in B_4. Similarly at each of the other apexes the exterior force may be resolved into components in the two given directions. Thus S_1, S_2, S_3, and S_4 are found as the stresses in B_1, B_2, B_3, and B_4.

To find the character of these stresses it is only necessary to follow around the sides of each force triangle in the direction indicated by the given force and then to transfer these directions to the corresponding apex of the frame. Thus, at the apex *n* the direction of P_2 is known and the corresponding force triangle is *bco*; in this P_2 acts from *b* toward *c*. hence S_2 acts from *c* toward *o* and S_1 acts from *o* toward *b*; transferri these directions to *n* it is found that S_1 acts toward and S_2 away from *n*, thus showing S_1 to be compression and S_2 to be tension. In like manner it is found that S_3 and S_4 are also tension.

Prob. 7. If the frame in Fig. 7 be inverted, draw the force diagram and determine the character of the stress in each member.

Prob. 8. In Fig. 7 let each of the forces P_1, P_2, P_3, and P_4 be vertical and equal to 100 pounds. Let B_1 and B_2 be horizontal, the length of the former being 6 feet and that of the latter being 2 feet. Let the lengths of B_3 and B_4 be 5 feet. Draw the force diagram and find the magnitude and character of the stress in each member.

ART. 6. THE EQUILIBRIUM POLYGON.

When a number of forces acting upon a body do not meet in a common point the magnitude and direction of their resultant is found by the closing line of the force polygon (Art. 3), but its line of action is not determined. This will now be found by means of another diagram which is called the equilibrium polygon.

Let four forces represented by P_1, P_2, P_3, and P_4 be given in magnitude, direction, and line of action, and let it be required to fully determine their resultant. Constructing the force polygon *abcde*, the length of the closing line *ea* represents the magnitude of the resultant, and its direction is from *a* toward *e*, being opposite to those of the other forces in following around the polygon (Arts. 2 and 3). Now select any point *o* and draw the lines *oa*, *ob*, *oc*, *od*, and *oe* to the vertices of the force polygon, thus forming five force triangles. In the force triangle *oab* the lines *oa* and *ob* represent two forces which can hold *ab* in equilibrium if their directions be from *b* to *o* and from *o* to *a*. Thus each of the forces in the force polygon can be replaced by its components shown by the broken lines; for example, P_2 has the components S_1 and S_2. Now through any point *m* on the line of action of P_1 draw the lines B_1 and B_5 parallel to S_1 and S_5 respectively, and let B_1 intersect the line of action of the force P_2 at *n*. Through *n* draw B_2 parallel to S_2, and so on in succession, until finally B_4 is drawn parallel to S_4. The lines B_4 and B_5 will intersect at some point *r*; through this point draw a line *R* parallel to *ae* and the line of action of the resultant is determined. For, by the construction the forces S_4 and S_5 are the components of the resultant *R* or *ae*, and as their lines of action are in B_4 and B_5 the resultant must pass through the point where they intersect.

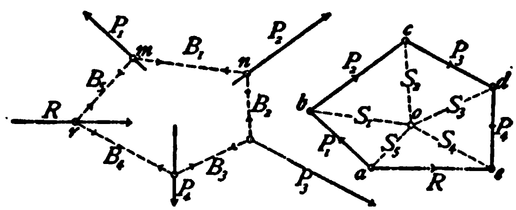


Fig. 8.

If in Fig. 8 there be applied at the point *r* a force P_5 equal to *R* but opposite in direction the forces P_1, P_2, P_3, P_4, and P_5 are in equilibrium and the force polygon closes. The polygonal frame $B_1B_2B_3B_4B_5$ thus holds the given forces in equilibrium by means of the stresses of tension and compression acting in its members. For the case shown in the figure these stresses are all tensile, and their values are given by the lines S_1, S_2, etc., in the force polygon. The lines of this frame are hence called an 'equilibrium polygon.' The polygonal frame in Fig. 7 is an equilibrium polygon which holds the exterior forces in balance.

The graphic condition of equilibrium for several forces not meeting at the same point may now be expressed by saying that both the force polygon and the equilibrium polygon must close. If the former closes and the latter does not the given forces are not a system in equilibrium. For example the three forces P_1, P_2, and P_3 in Fig. 9 are equal in magnitude and make angles of 120 degrees with each other, but they are not in equilibrium because their lines of action do not intersect in the same point. The force polygon *abc* here closes. Select any point *o* and draw the lines *oa*, *ob*, and *oc*, thus resolving the force P_1 into the components S_1 and S_2, the force P_2 into S_2 and S_3, and the force P_3 into S_3 and S_1. Now select any point *m* on the line of action of P_1 and draw *mr* and *mn* parallel to S_1 and S_2; from *n*, where *mn* intersects the line of action of P_3, draw *nr* parallel to S_3. Then *mr* and *nr* intersect at *r* which is not on the line of action of P_2, and hence the three given forces cannot be held in equilibrium by an equilibrium polygon. In this case it is said that the equilibrium polygon does not close. If, however, the force P_2 be moved parallel to itself until its line of action passes through *r* the force polygon closes and the forces will be in equilibrium.

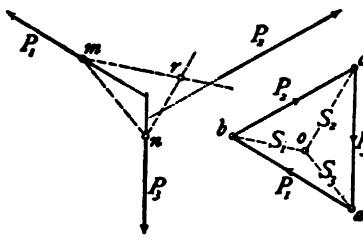


Fig. 9.

The point *o* in the plane of the force polygon is called 'the pole,' and the lines *oa*, *ob*, etc., are sometimes called 'rays.' Since the position of the pole may be selected at pleasure it follows that for any given system of forces an infinite number of equilibrium polygons can be constructed. The pole *o* may be taken either within or without the force polygon as may be most convenient for the solution of the problem under consideration.

Prob. 9. Given two forces of 100 and 180 pounds acting at an angle of 5 degrees with each other, the point of intersection not being within the limits of the drawing. Find the magnitude and direction of the resultant by the force polygon, and its line of action by the equilibrium polygon.

ART. 7. PROPERTIES OF THE EQUILIBRIUM POLYGON.

Let Fig. 10 represent a system of forces P_1, P_2, . . . P_6 held in equilibrium by the jointed frame or equilibrium polygon whose members are B_1, B_2, . . . B_6. This is constructed by first

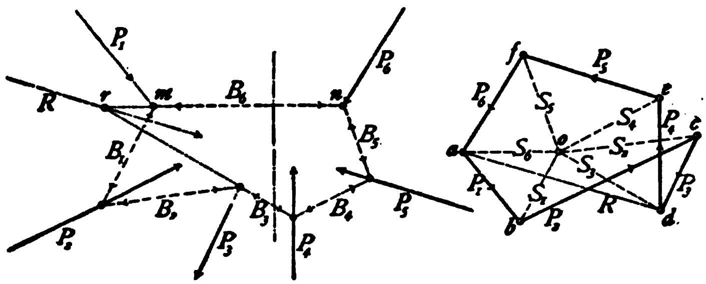


Fig. 10.

drawing the force polygon *abcdef* which must close, then selecting a pole *o* and drawing the rays *oa, ob,* . . . *of,* to which the members of the equilibrium polygon are made respectively parallel. The character of the stresses in these members is determined from the force polygon; thus in the triangle *abo* the directions of *bo* and *oa* must be from *b* to *o* and from *o* to *a* in order to maintain equilibrium; transferring these directions to the other diagram, it is seen that the stresses in B_6 and B_1 act toward the apex *m,* and hence are compression. Passing next to the vertex *n* the stress in B_5 is also found to be in compression, and so on. (Art. 4.)

Let this equilibrium polygon be cut by a section shown by the broken and dotted line; the stresses in B_3 and B_6, the members cut, are given by S_3 and S_6 in the force diagram, and form the closing sides of the polygon *abcdo* and also of the polygon *defao.* That is, the stresses in B_3 and B_6 hold in equilibrium the external forces P_1, P_2, and P_3, and also the external forces P_4, P_5, and P_6. Therefore the following principle is established:

> The internal stresses in any section hold in equilibrium the external forces on either side of that section.

This principle, it will be observed, is the same as that applicable to the internal stresses in a beam (Mechanics of Materials, Art. 15) or to the internal stresses in a truss (Roofs and Bridges, Part I, Art. 4).

Since the stresses in B_3 and B_4 hold in equilibrium the external forces P_1, P_2, and P_3 the resultant of the former must be equal and opposite to the resultant of the latter. This is also seen in the force polygon where the line *ad* gives the magnitude of this resultant. To find its line of action let B_3 and B_4 be produced until they meet in *r*, and through *r* draw a line equal and parallel to *ad*. Thus results the following important principle:

> The resultant of the external forces on the left of any section passes through the intersection of the sides of the equilibrium polygon cut by that section, its magnitude and direction being given by the force polygon.

The resultant of the external forces on the right of the section is the same in magnitude and line of action as that of those on the left, but its direction is reversed.

When a system of parallel forces are in equilibrium, the force polygon becomes a straight line and the equilibrium polygon has an important special property which will now be deduced. Let P_1, P_2, and P_3 in Fig. 11 be three downward forces, held in equilibrium by the two upward forces P_4 and P_5; for example, the former might be loads acting on a beam and the latter the reactions of the supports. The force polygon here is *abcdea*, the lines *ab*, *bc*, and *cd* being laid off downward while *de* and *ea* are laid off upward, closing the polygon. Selecting any pole *o*, and drawing the rays *oa*, *ob*, etc., the equilibrium polygon B_1B_2 $B_3B_4B_5$ is formed (Art. 6). Now let any two sides B_1 and B_4 be cut by a vertical plane, and let the ordinate intercepted between them be called *y*. The intersection of these sides produced gives the point of application of the resultant of the external forces P_1 and P_2, whose value is given by *eb* in the force polygon; let the horizontal distance from *y* to this point be called *r*, and the resultant be called *R*. The bending moment

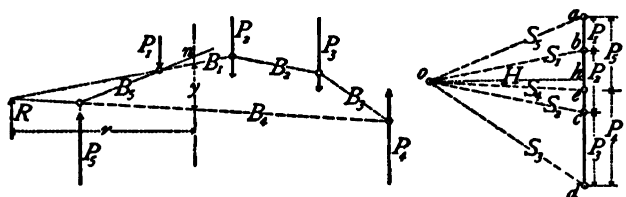


Fig. 11.

M in the given section is then equal to the moment of the resultant of all the forces on the left of that section, or

$$M = Rr.$$

Now in the force polygon let the line *oh* be drawn horizontally through the pole and its value be called *H*. Then since the triangle *obe* is similar to the triangle which has the base *y* and the altitude *r*, and since *eb* is equal to *R*, we have

$$r : y :: H : R \qquad \text{or} \qquad Rr = Hy.$$

Therefore the bending moment of the external forces on the left of the section is

$$M = Hy.$$

The force *H* is seen to be the horizontal component of each of the stresses *oa*, *ob*, etc., in the force polygon, that is, of the stresses in the members B_1, B_2, etc., in the equilibrium polygon; it is called the 'pole distance,' and is measured by the same scale of force as the other lines in the force polygon. The following theorem can hence be stated:

> If a structure be subject to parallel forces, the bending moment in any section parallel to the forces is equal to the ordinate *y* in the equilibrium polygon multiplied by the pole distance *H* in the force polygon.

Hence by adopting suitable scales the values of the bending moments can easily be found from the diagram. For instance, if the linear scale used in laying off the positions of the external forces be 20 feet to the inch, and if the pole be so selected that the distance H is 5 000 pounds, then the moments are are measured by a scale of 100 000 pound-feet to the inch.

The following is another proof of this important theorem: Let the line B_4 in Fig. 11 be produced until it meets the section in n; let the vertical distance between n and B_4 be called y_4, and that between n and B_1 be called y_1. Let the lever arms of P_1 and P_4 with respect to the section be called p_1 and p_4. The bending moment for the section then is

$$M = P_4 p_4 - P_1 p_1 .$$

But the triangle whose base is y_4 and altitude p_4 is similar to *oae*, and the triangle whose base is y_1 and altitude p_1 is similar to *oab*. Hence $P_4 p_4 = Hy_4$ and $P_1 p_1 = Hy_1$; and accordingly

$$M = H(y_4 - y_1).$$

But since $y_4 - y_1 = y$, this gives $M = Hy$, which is the same relation as before deduced.

Prob. 10. Draw an equilibrium polygon for the five vertical forces given in Fig. 11, taking the pole on the right-hand side of the force polygon.

Prob. 11. Given two parallel forces 12 feet apart and acting in opposite directions, one being 8 000 pounds and the other 3 000 pounds. Find by the force and equilibrium polygons the magnitude and line of action of their resultant.

ART. 8. REACTIONS OF BEAMS.

By the use of the force and equilibrium polygons the reactions of the two supports of a beam carrying given loads may be graphically determined. For example, let the beam in Fig. 12 be subject to two concentrated loads as shown, and be in equilibrium under the action of these loads and the two reactions. If the values of the reactions were known an equilibrium polygon could then be constructed which should act instead of the beam to maintain this equilibrium (Art. 7). But since the loads and reactions constitute a system of forces in equilibrium, the principle that the equilibrium polygon must close (Art. 6) furnishes the means of determining the unknown reactions.

Let P_1 and P_2 be the given loads, and let *ab* and *bc* be drawn equal and parallel to them. Since the force polygon must close the line *ca* then represents the sum of the two reactions. Next let any pole *o* be selected, and the rays *oa*, *ob*, and *oc* be drawn, and parallel to these let B_4, B_1, and B_2 be drawn, thus forming part of the equilibrium polygon. This polygon can

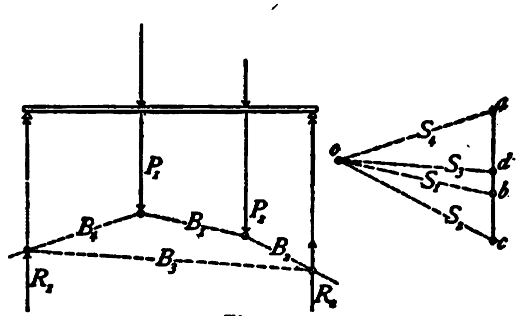


Fig. 12.

now be closed by drawing B_3, joining the points where B_4 and B_2 intersect the lines of action of the reactions. Finally, in the force polygon let *od* be drawn through *o* parallel to B_3; thus determining the point *d*; then *cd* and *da* are the two reactions, the former being R_2, and the latter R_1. For, *cd* is the force that holds in equilibrium the stresses S_2 and S_3 in the members B_2 and B_3, and *da* is the force that holds in equilibrium the stresses S_3 and S_4 in B_3 and B_4.

As a second example, let it be required to determine the reactions for the overhanging beam shown in Fig. 13 due to the two given loads. Laying off *ab* and *bc* as before, the sum of the reactions is shown by the line *ca*. Choosing a pole *o* and drawing *oa*, *ob*, and *oc*, the equilibrium polygon is constructed by taking any point on the line of action of P_1 and drawing B_4 and B_1 parallel to *oa* and *ob* respectively. Then from the point where B_1 intersects the line of action of P_2 the line B_2 is drawn parallel to *oc*. Now B_4 intersects the line of action of R_1 at *m* and B_2 intersects that of R_2 at *n*; joining *mn* by the line B_5 closes the equilibrium polygon, and parallel to this closing line *od* is to be drawn in the force diagram, thus determining the two reactions *cd* and *da*.

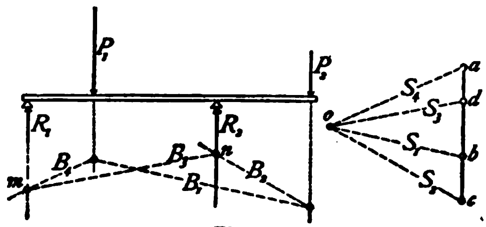


Fig. 13.

In both the above figures, any ordinate drawn in the equilibrium polygon gives the bending moment in the beam at the point vertically above it (Art. 7). In Fig. 13 where the sides of the equilibrium polygon cross there is no ordinate, and this corresponds to the position of the inflection point in the beam where the horizontal stresses change from tension to compression.

Prob. 12. Given an overhanging beam as in Fig. 13, its length being 18 feet, and the distance between the supports 14 feet. Determine the reactions due to three loads, one of 200 pounds at 3 feet from the left end, one of 80 pounds at 6 feet from the left end, and one of 90 pounds at the right end.

ART. 9. SIMPLE BEAMS UNDER CONCENTRATED LOADS.

By applying the principles of the preceding articles the vertical shears and the bending moments may be found for all sections of a beam having only two supports and subject to any number of concentrated loads. For example, consider a simple beam 20 feet long, carrying five loads whose positions, and weights in pounds, are shown in Fig. 14. The reactions of the supports are found by laying off the loads successively on the force polygon, or load line *af*, the first load being *ab*, the second *bc*, etc. Select the pole *o*, draw the rays from *o*, and construct the equilibrium polygon *m*, *n*, etc., its closing line being *ms*. Then through *o* draw *og* parallel to *ms*, and the lines *fg* and *ga* will represent the reactions of the right and left supports.

Between the left support and the first load the vertical shear equals the reaction *ga*; between the first and second loads the vertical shear is $ga - ab = gb$; between the second and third

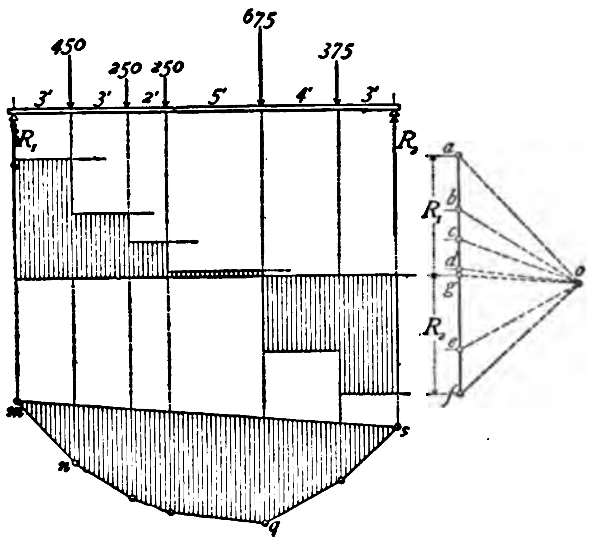


Fig. 14.

loads it is $ga - ab - bc = gc$; and so on. At the fourth load the shear changes from positive to negative, and at the right support its value is the reaction *gf*. The diagram shown in the figure above the equilibrium polygon gives these shears, and the manner of its construction is apparent, each step being one of the loads. This is called the shear diagram.

The ordinates in the equilibrium polygon, or moment diagram, give the bending moments in the corresponding sections of the beam. It is seen that the maximum ordinate is where the sides of the equilibrium polygon meet that are parallel to *od* and *oe*, the rays on opposite sides of and adjacent to *og*, the line parallel to the closing side *ms* of the equilibrium polygon. As the shear immediately on the left of the corresponding section is positive, being measured from *g* to *d*, and that on the right is negative, being measured from *g* to *e*, the important relation is obtained that the maximum bending moment occurs at the section where the vertical shear passes through zero.

In making the actual construction for Fig. 14 the linear scale used in laying off the beam and the positions of the load was 5 feet to an inch, and the force scale used in the force polygon was 800 pounds to an inch. The pole distance was taken as 1 000 pounds, and hence the moment scale was 5 000 pound-feet to an inch. Any ordinate in the shear diagram, measured by the force scale, gives the vertical shear in pounds; thus, between the second and third loads the shear is + 300 pounds, and between the fourth and fifth loads it is − 625 pounds. Any ordinate in the moment diagram, measured by the moment scale, gives the bending moment in pound-feet; thus, the maximum bending moment is 5 500 pound-feet. Fig. 14, however, as here printed, is about one-half the size of the actual construction.

Prob. 13. Construct the shear diagram and moment diagram for a beam 16 feet long, carrying two loads, each of 4 000 pounds, one being at 5 feet from the left end, and the other at 5 feet from the right end.

ART. 10. SIMPLE BEAMS UNDER UNIFORM LOADS.

Let a simple beam whose span is l be uniformly loaded with the weight w per linear unit; then each reaction is equal to half the total load or $\frac{1}{2}wl$. The load may be represented graphically by the shaded rectangle on the beam whose base is l and altitude w.

For any section at a distance x from the left support the vertical shear is $V = \frac{1}{2}wl - wx = w(\frac{1}{2}l - x)$; if V be an ordinate corresponding to an abscissa x this is the equation of a straight line. Thus when $x = 0$, $V = \frac{1}{2}wl$; when $x = \frac{1}{2}l$, $V = 0$; and when $x = l$, $V = -\frac{1}{2}wl$. The shear diagram is hence constructed by laying off gi equal to the span, making gf and ik equal to $\frac{1}{2}wl$ and joining f with k.

The bending moment in a section distant x from the left support is $M = \frac{1}{2}wl \,.\, x - wx \,.\, \frac{1}{2}x = \frac{1}{2}w(lx - x^2)$. This is the equation of a parabola; for $x = 0$ and $x = l$ the value of M is 0; for $x = \frac{1}{2}l$, M reaches its maximum value $\frac{1}{8}wl^2$. The moment diagram may hence be constructed by laying off mn equal to the span, drawing qr at the middle equal to the maximum moment, and then constructing the parabola mrn. To do this the lines ms and ns are drawn, rs being made equal to qr, these are divided into the same number of equal parts and the points of division joined as shown, thus determining tangents to the parabola.

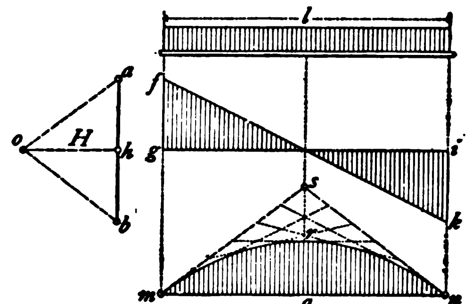


Fig. 15.

If the entire load on the beam were concentrated at the middle, aba would be the force polygon, and bh and ha the two reactions. Now let o be a pole having the pole distance H, and let the equilibrium polygon msn be constructed. Then from the similar triangles oah and msq,

$$H : \tfrac{1}{2}wl :: \tfrac{1}{2}l : qs.$$

Hence if H be equal to unity on the scale of force, the ordinate qs has the value $\frac{1}{4}wl^2$, and since qr is $\frac{1}{8}wl^2$ the maximum moment for a single concentrated load at the middle is twice as great as that due to the same load when uniformly distributed.

Prob. 14. Prove that the sum of all the moments due to a uniform load is two-thirds of the sum of all the moments due to the same load when concentrated at the middle.

ART. 11. OVERHANGING BEAMS.

Let a beam be taken with one overhanging end and bearing a number of concentrated loads as shown in Fig. 16. The loads are given in pounds and the distances in feet. If in the force polygon the loads be laid off successively in the order in

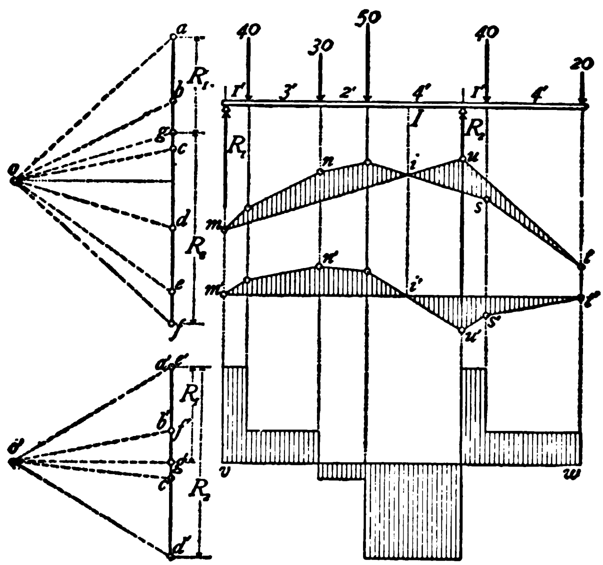


Fig. 16.

which they are on the beam, and the equilibrium polygon *m . . n . . stu* be constructed, the ray *og* drawn parallel to the closing line *mu* will determine the reactions *fg* and *ga*. The sides of the equilibrium polygon are found to cross each other at *i*, and the ordinates to the right of this point lie on the opposite side of the closing line from those on the left. The ordinates on the left being regarded as positive, those on the right are negative, and they give the bending moments for all sections in the beam. The point I, where the bending moment is zero, is called the inflection point; on the left of this point the lower fibers are in tension while on the right they are in compression.

In order to construct a moment diagram whose ordinates shall be measured from a horizontal line $m't'$, the numerical values of the reactions should first be computed; these are $R_1 = 60$ pounds and $R_2 = 120$ pounds. Now form a force polygon, or load-line, for the reactions and loads by laying off the reaction R_1 from g' upward to a', then the first three loads in succession from a' downward to d', then the reaction R_2 from d' upward to e', and finally the remaining loads from e' downward to g'. Take the pole o' on a perpendicular to the load line at g'. The equilibrium polygon $m' \,..\, n' \,..\, u's't'$ can then be constructed, each of whose ordinates is equal to that in the polygon $m \,..\, n \,..\, stu$.

The method of constructing the shear diagram on the axis vw will be understood without further explanation than that given in Art. 9. It is seen that the shear passes through zero at two points, one where the maximum positive moment occurs, and the other at the right support where the negative moment is a maximum.

The linear scale used in the actual construction of Fig. 16 was 4 feet to an inch, and the force scale was 60 pounds to an inch; the pole distance being 100 pounds, the moment scale was 400 pound-feet to the inch. In the figure as printed the scales are one-half these values. By measurement it is found that the maximum shear is 60 pounds, the maximum positive moment 120 pound-feet, and the maximum negative moment 140 pound-feet.

For the case of a uniform load a shear diagram and moment diagram may be constructed by computing the maximum ordinates and then drawing the straight lines and parabolas (Mechanics of Materials, Chap. IV). Thus, let Fig. 17 represent a beam 28 feet long with ends overhanging 4 and 6 feet, and let the uniform load be 40 pounds per linear foot. The left reaction is found by computation to be 497.8, and the right reaction to be 622.2 pounds; these might also be obtained graphically by the equilibrium polygon, regarding the load on each portion of the beam as concentrated at its center. The shear is then found to be zero at c, distant 8.45 feet from the left support, and at this point the positive moment is a maximum, its value by computation being 1 106 pound-feet. At the left support the negative moment is found to be 320 and at the right support 720 pound-feet. These moments being laid off by scale the curves can be constructed by the method given in Art. 10, it being known that the end parabolas have their vertices at m' and q, and that the middle parabola has its vertex at n. The inflection points are equally distant from the place of maximum positive moment, this distance being 7.45 feet in Fig. 17. The diagrams thus furnish full information regarding the distribution of the shears and moments in the beam.

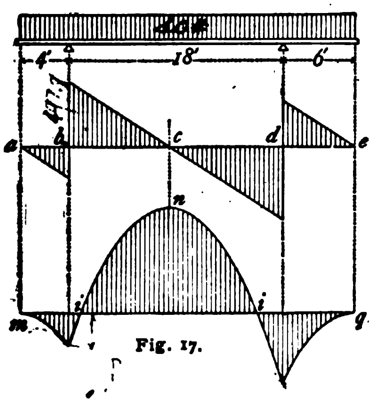


Fig. 17.

Prob. 15. A beam 20 feet long has two overhanging ends, each 5 feet long. Draw the shear and moment diagrams due to a load of 800 pounds at one end and a load of 500 pounds at the other end.

Prob. 16. A beam 20 feet long has two overhanging ends, each 5 feet long. Draw the shear and moment diagrams due to three loads, one of 800 pounds at one end, one of 500 pounds at the other end, and one of 1 000 pounds at the middle.

ART. 12. CENTER OF GRAVITY OF CROSS-SECTIONS.

In problems relating to the strength of beams it is necessary to find the position of the neutral axis of the cross-section (Mechanics of Materials, Chap. III). The neutral axis passes through the center of gravity of the cross-section, and in finding the position of the center of gravity the equilibrium polygon may be employed. For instance, let the cross-section of a deck beam shown in Fig. 18 be taken. As this cross-section has an axis of symmetry *AD*, the center of gravity lies on this axis. The area is divided into simple geometrical figures or narrow strips by lines perpendicular to the axis, and at the centers of gravity of these parts forces proportional to their areas are applied. The force and equilibrium polygons are constructed in the usual manner, taking the pole opposite the center *h* of the force line and making the pole distance *oh* equal to *ah*. The extreme sides of the equilibrium polygon *m* . . *n* . . *q* are produced until they meet at *r*, the special position of the pole causing them to form the best intersection, that is, at right angles. According to Art. 6, the point *r* is in the line of action of the resultant of the forces considered, hence *rC* drawn parallel to the forces is the line of action of the resultant. The center of gravity lies on this line and therefore at its intersection with the axis *AD*.

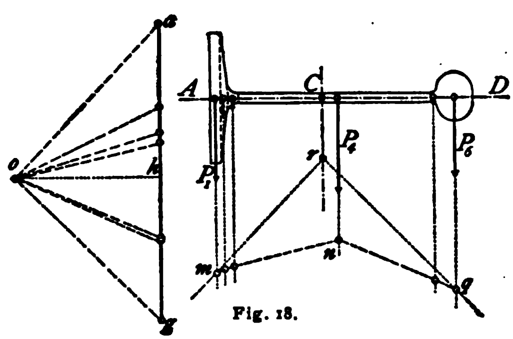


Fig. 18.

If the surface is very irregular in outline it should be divided into strips so narrow that the area of each one is equal to the product of its mean length by its width without appreciable error. If there be no axis of symmetry, the process described above must be repeated for a direction at right angles to the first, and the center of gravity will lie at the intersection of the two resultants found.

Prob. 17. Find the center of gravity of the cross-section of the channel-iron whose dimensions in inches are given in Fig. 19.

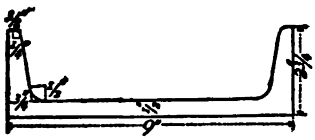


Fig. 19.

Prob. 18. Five parallel forces act in the same direction. Beginning at the left their magnitudes are 45, 120, 30, 225, and 80 pounds, and the successive distances between them are 6, 10, 3, and 8 feet. Determine the magnitude and position of their resultant.

ART. 13. MOMENT OF INERTIA OF CROSS-SECTIONS.

For beams under flexure the bending moment M for any section equals the resisting moment $\frac{SI}{c}$ with reference to the neutral axis in that section in which S is the unit-stress in the most remote fiber distant c from the axis and I is the moment of inertia of the cross-section with reference to the same axis (Mechanics of Materials, Art. 19).

In Art. 12 the method is given for finding the center of gravity through which the neutral axis passes, and the moment of inertia I may be obtained from the same construction by the following rule: Let A be the area of the given cross-section, and A' the area included between the equilibrium polygon and the two lines whose intersection determines the center of gravity; then I is the product of A by A', or $I = AA'$, provided that the pole distance is $\frac{1}{2}A$. This rule will now be demonstrated in connection with a practical example.

Let it be required to find I for the T-shaped section which is shown in Fig. 20. The cross-section A measures 4.04 square inches, and by the force and equilibrium polygon the neutral axis EC is found to be 1.32 inches from the base of the T. Now produce sq until it meets the axis at t. The triangles qtu and ofe are similar, as their sides are mutually parallel. Let y be the distance from q to the axis; then

$$tu : y :: ef : oh.$$

But ef equals the area P_5 laid off to scale, and the pole distance oh was made equal to $\frac{1}{2}A$; hence,

$$tu \,.\, \tfrac{1}{2}A = P_5 y.$$

Multiplying this by y, and remembering that $\frac{1}{2}tu \,.\, y$ is the area of the triangle qtu, gives,

$$\text{area } qtu = \frac{P_5 y^2}{A}.$$

The other triangles composing the area between the equilibrium polygon and the lines mr and sr may be expressed in a similar manner. If the area P_5 were of the width dy its moment of inertia would be $P_5 y^2$, and by adding all the areas together there would be found,

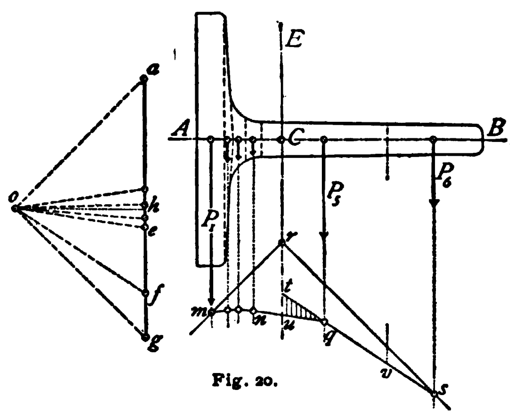


Fig. 20.

$$A' = \frac{I}{A}, \qquad \text{or} \qquad I = AA',$$

in which case the broken line $m \ldots nuqs$ becomes a curve which is tangent to the lines mr and sr at the extreme limits of the given cross-section. This curve may be drawn and the area A' determined either by dividing it into strips or by the planimeter.

By performing the above operations on a full scale drawing for the T iron shown in Fig. 20 the area A' was found by the planimeter to be 1.80 square inches, hence the moment of inertia is

$$I = 4.04 \times 1.80 = 7.27 \text{ inches}^4.$$

This agrees very well with the value given in the manufacturer's pocket-book, which is 7.26 inches^4.

Prob. 19. Determine the moment of inertia of the channel iron shown in Fig. 19 with respect to an axis through the center of gravity and normal to the web. Determine also the moment of inertia for an axis through the center of gravity and parallel to the web.

ART. 14. GRAPHICAL ARITHMETIC.

In the various operations that are performed on the drawing board cases occur where quantities are to be added, subtracted, multiplied, or divided, and it frequently happens that this may be done by some simple graphical procedure instead of by the common methods of arithmetic. Whatever be the nature of these quantities they are represented by straight lines, the number of units in the length of a line denoting the magnitude of the quantity.

To add together several lines it is only necessary to place them end to end and then measure the total length by the scale.

To subtract one line from another the first is to be laid off from the end of the second in the opposite direction and then their difference can be measured by the scale.

To add algebraically several quantities, some positive and some negative, the sum of the lines representing the negative ones is to be found and subtracted from the sum of the lines representing the positive ones.

To multiply any line of length a by any quantity represented by a line of length m, let OX and OY be drawn making a convenient angle with each other. Lay off Ob equal to unity, Oa equal to a, and Om equal to m. Then Oy is the required product $a \times m$. For, by similar triangles,

$$Oy : Om :: Oa : Ob,$$

whence
$$Oy = Oa \times Om = a \times m.$$

By successive applications of this principle the product of several factors may be found.

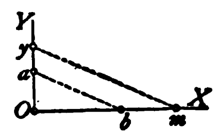


Fig. 21.

To divide a line of length m by one of length b, let Oa be laid off equal to unity, Om equal to m, and Ob equal to b; then making the construction as before, the similar triangles give

$$Oy = Om \div Ob = m \div b,$$

and hence Oy is the quotient found by dividing m by b.

By the help of the principle of similar triangles a number of graphical constructions can be made for adding and multiplying fractions. For instance, let it be required to find the product of the two fractions $\frac{6}{13}$ and $\frac{9}{8}$. On any line OX lay off, as in Fig. 22, $Oa = 13$ and $Oc = 8$; erect the ordinates $ab = 6$ and $cd = 9$; join Ob and Od. Select any abscissa Ox, in this case taken as 10, draw the ordinate xy, and taking Ox' equal to xy, draw the ordinate $x'y'$. Then the product of the two fractions is one-tenth of $x'y'$. For, by similar triangles,

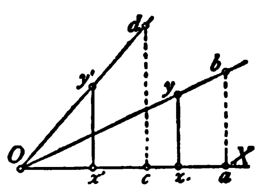


Fig. 22.

$$\frac{ab}{Oa} = \frac{xy}{Ox} \quad \text{and} \quad \frac{cd}{Oc} = \frac{x'y'}{Ox'}.$$

Multiplying these together, term by term, and inserting the values, there results

$$\frac{6}{13} \times \frac{9}{8} = \frac{x'y'}{10}.$$

Measuring $x'y'$ by the scale it is found to be 5.2; hence the required product is 0.52.

Prob. 20. Show by Fig. 21 how to graphically transform the fraction $\frac{2.5}{5.5}$ to an equivalent fraction having 9 for its denominator.

Prob. 21. Make the following graphical constructions: (*a*) to square a given number, (*b*) to extract the square root, (*c*) to find the reciprocal, the final result in each case to be represented by the length of a straight line.

CHAPTER II.

ROOF TRUSSES.

ART. 15. DEFINITIONS AND PRINCIPLES.

A roof truss is a structure whose plane is vertical, and is supported at its ends by the side walls of the building, being so arranged that its principal members are subject only to tensile or compressive stresses under the influence of the loads which it is designed to carry.

The points where the center lines of the adjacent members meet are the centers of the connections which form the joints. The joints of the truss are supposed to be perfectly flexible, and the external forces, consisting of the loads and reactions, to be applied only at the joints.

For stability the elementary figures composing a truss must be triangles, since a triangle is the only polygon which cannot change its shape without altering the lengths of its sides when loaded at one or more joints.

The 'span' of a truss is the distance between the end joints or the centers of the supports, and the 'rise' is the distance from the highest point, or peak, to the line on which the span is measured.

The 'upper chord' consists of the upper line of members extending from one support to the other. Each half of the upper chord of a triangular truss is sometimes designated the 'main rafter.' The lower line of members is known as the 'lower chord' or 'tie rod.'

The 'web members' or 'braces' connect the joints of the upper with those of the lower chord, and may be either verticals, diagonals, or radials.

A member which takes compression is called a 'strut,' and one that takes tension a 'tie.' The upper chord and some of the braces are subject to compression while the lower chord and the rest of the braces are in tension.

The fundamental principles of Graphic Statics as given in Chapter I apply to the determination of the stresses in trusses under given conditions of loading, in the manner indicated in Art. 4. The notation employed in this chapter for designating truss members and loads differs from that previously used, the letters being placed upon spaces instead of upon lines, and any member is named by the letters between which it is situated. Thus, in Fig. 23, AC and BC are the two rafters and CD is the tie rod, while AB designates the load at the peak.

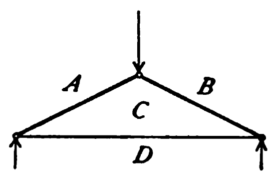


Fig. 23.

Prob. 22. The span in the simple triangular roof truss of Fig. 23 is 26 feet, and the rise is 6 feet 6 inches. Find the stresses in the members due to a load at the peak of 2 050 pounds.

ART. 16. DEAD AND SNOW LOADS.

Four kinds of loads are to be considered in discussing a truss: the weight of the truss itself, the weight of roof covering, the snow, and the wind.

The weight of the truss depends upon its span and rise, the distance between adjacent trusses, the kind of material used in the construction of the roof covering and the truss, and other elements of design. This weight is ascertained from the results of experience, designs being made for various spans and then the weights of the trusses computed. As rough approximate formulas for use in computing stresses the following may be employed for finding the weight of roof trusses not greater in span than 200 feet.

For wooden trusses, $W = \frac{1}{2}al(1 + \frac{1}{10}l)$,
For wrought-iron trusses, $W = \frac{3}{4}al(1 + \frac{1}{10}l)$,

in which l is the span in feet, a the distance in feet between adjacent trusses, and W the approximate weight of one truss in pounds. The wooden trusses are to have wrought iron tension members in accordance with the usual practice. It is seen that they are one-third lighter than the wrought iron trusses.

The roof covering consists of the exterior 'shingling' of tin, slate, tiles, corrugated iron, or wooden shingles, resting usually upon timber 'sheathing,' which is supported by 'purlins,' or beams running longitudinally between the trusses and fastened to them at the upper joints. In large roofs the sheathing is laid upon 'rafters' parallel to the upper chord, the rafters resting upon the purlins. The actual weight of the roof covering, rafters, and purlins is to be determined only by computation for each particular case, but the following values will serve for preliminary designs and approximate computations. The weights given are in all cases per square foot of roof surface.

For shingling—tin, 1 pound; wooden shingles, 2 or 3 pounds; iron, 1 to 3 pounds; slates, 10 pounds; tiles, 12 to 25 pounds.
For sheathing—boards 1 inch thick, 3 to 5 pounds.
For rafters—1.5 to 3 pounds.
For purlins—wood, 1 to 3 pounds; iron, 2 to 4 pounds.
Total roof covering—from 5 to 35 pounds per square foot of roof surface.

The snow load varies with the latitude, being about 30 pounds per horizontal square foot in Northern New England, Canada, and Minnesota, about 20 pounds in the latitude of New York City and Chicago, about 10 pounds in the latitude of Baltimore and Cincinnati, and rapidly diminishes southward. On roofs having an inclination to the horizontal of 60 degrees or more this load may be neglected, as it might be expected that the snow would slide off.

The weight of a roof truss with that of the roof covering which it bears is termed the 'dead load' or 'permanent load.'

For the purpose of securing uniformity in the solution of the examples and problems given in this book, the following average values will be used, unless otherwise specified:

For the truss weight—compute from the above formulas.

For the roof covering—12 pounds per square foot of roof surface.

For the snow load—15 pounds per square foot of horizontal area.

The weight of the roof covering, and of the snow which may be upon it, is brought by the purlins to the joints or 'apexes' of the upper chords. The weight of the truss itself is also generally regarded as concentrated at the same points, the larger part of it being actually applied there. At each apex of the upper chord there is hence a load called an 'apex load,' and it may be a 'dead apex load' or a 'snow apex load.'

In the wooden truss whose outlines are shown in Fig. 24, let the span be 48 feet, the rise 10 feet, and the distance between the trusses 12 feet. Then the length of the rafter is $\sqrt{24^2 + 10^2} = 26$ feet. Each main rafter is divided into three equal parts called 'panels.' From the formula the truss weight is found to be 1 670 pounds. The weight of the roof covering on each rafter

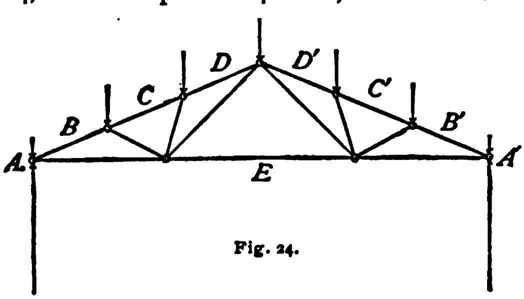


Fig. 24.

is $26 \times 12 \times 12 = 3\,744$ pounds. The weight of the snow supported by the entire truss is $48 \times 12 \times 15 = 8\,640$ pounds. The total dead load is $1\,670 + 2 \times 3\,744 = 9\,158$ pounds, and the dead apex loads *BC*, *CD*, *DD'*, *D'C'*, and *C'B'* are each one-sixth of the total load or 1 526 pounds, while *AB* and *B'A'* are each one-half as much, or 763 pounds. The snow apex loads are in like manner 1 440 pounds and 720 pounds. The apex load is also called the 'panel load.'

If the panels be of unequal lengths, the load at any apex is found by considering that the weights brought to it by the purlins are those upon a rectangle extending in each direction half-way to the adjacent apexes.

When the two halves of the truss are symmetrical the reactions of the supports are equal, each being one-half of the total load. When the truss is unsymmetrical, the reactions are found in the same way as for concentrated loads on a beam. In the above example each dead load reaction is 4 579 pounds and each snow load reaction is 4 320 pounds.

The reaction and the half apex load acting at each support produce an effective reaction equal to their difference. This effective reaction is that due to the other apex loads, and therefore the half apex loads at the supports may be omitted entirely from consideration. The effective reaction for the above example is $4\,320 - 720 = 3\,600$ pounds, or by disregarding the loads at the supports it is one-half the sum of the full panel loads, thus, $\frac{1}{2}(5 \times 1\,440) = 3\,600$ pounds.

Prob. 23. A wrought iron truss, like Fig. 24, has a span of 60 feet, rise 14 feet, and distance between trusses 16 feet. Find the apex loads and reactions due to dead and snow loads.

Prob. 24. A wrought iron truss of the above form has a span of 90 feet 6 inches and a rise of 18 feet 9 inches, the trusses being 18 feet apart. Compute the apex loads and reactions.

ART. 17. STRESSES DUE TO DEAD AND SNOW LOADS.

The wooden truss shown in Fig. 25 has a span of 42 feet, rise at peak 14 feet, rise at hip 10 feet 3 inches, horizontal distance from hip to peak 12 feet 6 inches, and a distance between trusses of 8 feet.

The lengths of the members BG and EL are each $\sqrt{8.5^2 + 10.25^2} = 13.32$ feet, and of CH and DK $\sqrt{12.5^2 + 3.75^2} = 13.05$ feet. The weight of the truss is found to be 874 pounds by the formula in Art. 16. The roof covering on BG weighs $13.32 \times 8 \times 12 = 1\,279$ pounds, and on CH is $13.05 \times 8 \times 12 = 1\,253$ pounds. The snow load on BG is $8.5 \times 8 \times 15 = 1\,020$ pounds, and on CH is $12.5 \times 8 \times 15 = 1\,500$ pounds. The apex loads, expressed in pounds, are then as follows:

	$AB = EF$	$BC = DE$	CD	TOTAL
Truss, ...	109	218	218	872
Roof covering, . .	640	1 266	1 253	5 065
Dead, ...	749	1 484	1 471	5 937
Snow, ...	510	1 260	1 500	5 040

Each dead load reaction is therefore 2 969 pounds, and each snow load reaction 2 520 pounds.

The truss diagram, Fig. 25, composed of the center lines of the truss members, is carefully drawn to as large a scale as convenient, each joint being marked by a fine needle point and surrounded by a small circle to limit the lines drawn toward the point. In the actual construction the diagram was drawn to a scale of 3 feet to an inch, but the above figure is reduced to nearly one-seventh of the original size.

The external forces acting upon the truss are in equilibrium, and hence form a closed force polygon (Art. 2). These forces being parallel the resulting polygon becomes a straight line, which is called the 'load line.'

Taking first only the dead load and using a suitable scale (1 000 pounds to an inch was used on the original drawing), the apex loads taken in regular order from left to right are laid off in succession on the vertical load line *af* in Fig. 26, thus: the distance *ab* is laid off equal to the load *AB* or 749 pounds, then *bc* equal to the load *BC*, and so on. Next *fm* is laid off upward equal to the reaction *FM* or 2 969 pounds, and *ma* equal to *MA*, thus closing the polygon.

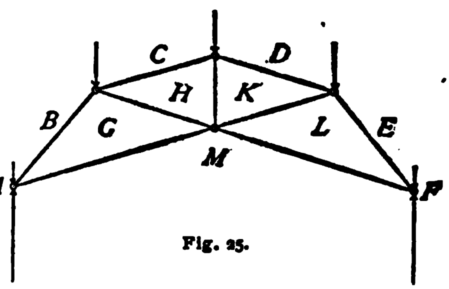


Fig. 25.

Beginning with the joint at the left support where the reaction *MA* and the load *AB* are held in equilibrium by the stresses in *BG* and *MG* the polygon representing these forces will also be closed; *bg* is therefore drawn parallel to *BG* and *mg* parallel to *MG*. The lengths of *bg* and *mg*, measured by the scale used on the load line, give the magnitudes of the stresses in these members. To find the character of the stresses the direction around the polygon indicated by the upward reaction is followed, that is, from *m* to *a*, *a* to *b*, *b* to *g*, and from *g* to *m*. Transferring these directions to the joint considered, the stress in *BG* acts toward the joint and is therefore compression, while that in *GM* acts away from the joint and is tension.

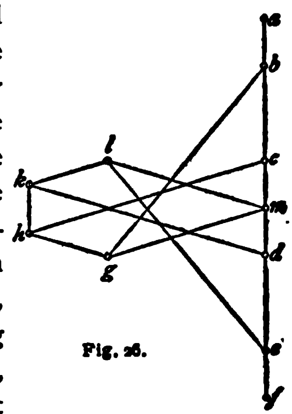


Fig. 26.

The stress diagram is continued by passing to the left hip-joint and constructing the force polygon *bchg* to represent the stresses in the members meeting at that joint. Next are constructed the polygons for the right support, the right hip-joint, and finally for the joint at the peak. The closing line *hk* must be parallel to *HK*, and the entire diagram be symmetrical with respect to a horizontal axis through *m*. These closed force polygons are drawn for all the joints taken in succession because the loads and stresses acting upon each joint in the truss are in equilibrium.

The points *l*, *k*, *h*, and *g*, in Fig. 26, may be regarded as the poles of the various equilibrium polygons which surround the spaces *L*, *K*, *H*, and *G*, in Fig. 25. After the application of the principles is fully understood, it is not necessary even to consider the entire polygon for any one joint. In determining the point *h*, for instance, which corresponds to the space *H*, it is found that the points *g* and *c* corresponding to the spaces *G* and *C*, that are adjacent to *H*, are already fixed in position, whence by drawing *ch* parallel to *CH* and *gh* parallel to *GH* the point *h* is obtained by their intersection.

The method given above for finding the character of the stresses requires passing around the perimeter of every force polygon contained in the stress diagram, unless the truss and its loading are symmetrical, in which case only one half of the polygons are so used. The line representing the stress in each member must in all cases be traced twice, and if no external force acts at any joint the direction of passing around the corresponding force polygon must be obtained from the character of the stress in one of its connected members already found. Thus, for the middle joint of the lower chord the stress in *GM* is known to be tension, and therefore acts away from this joint; that is, toward the left. The polygon *mghklm* must therefore be followed around in the direction indicated by the order of the letters here given.

Another method will now be described by which the character of the stress in any member may be determined without reference to that in any other member, and by which considerable time may be saved. For convenient reference

Figs. 25 and 26 are reprinted on this page, a slight addition being made to the latter as will be explained, and therefore it is marked Fig. 26*a*.

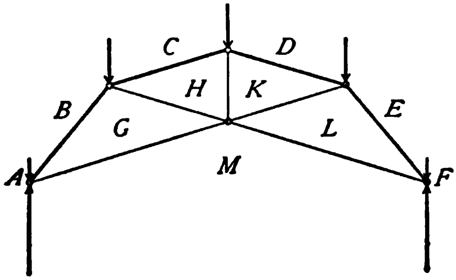


Fig. 25.

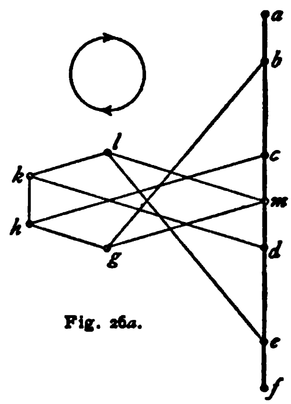


Fig. 26a.

The loads *AB*, *BC*, *CD*, *DE*, and *EF*, and the reactions *FM* and *MA*, were laid off on the load line in Fig. 26*a* in the order just stated; that is, they were taken in regular order while passing around the truss in Fig. 25 in the direction of the hands of a watch. As an aid in remembering this fact the circular arrow is placed aside of the stress diagram. By passing in the same direction around the left hip joint, for instance, the letters in the adjacent spaces occur in the order *C-H-G-B-C*. The corresponding force polygon has the same lower-case letters at its vertices, and is to be followed around so that its letters *c-h-g-b-c* shall also have the same order. When the directions thus indicated, *c* to *h*, *h* to *g*, etc., are transferred to the joint, *ch* acts toward it and is therefore compression, while *hg*, acting away from it, is tension, and *gb* is compression. Again, let it be required to find the kind of stress in *ML*. With reference to the middle joint of the lower chord these letters occur in the order *L-M*, hence the direction of its stress is from *l* to *m* in the stress diagram. As this direction is away from the joint, *LM* must be in tension.

As it is evident that trusses supported like that in Fig. 25 have the upper chord in compression and the lower chord in tension, whatever the arrangement of the web members may be, it usually remains to find only the kinds of stress in the web members. In this example they may all be found by considering the order of the letters in the web spaces with reference to the middle joint of the lower chord, and in any other truss having more web members by similarly taking the letters in their regular succession with reference to the respective joints of one of the chords. Let the student test this statement in the next article and observe that the web stresses will thus form a connected chain which may be rapidly traced. Let the joints of the lower chord be used first, and those of the upper chord afterwards, for a comparison of their relative convenience.

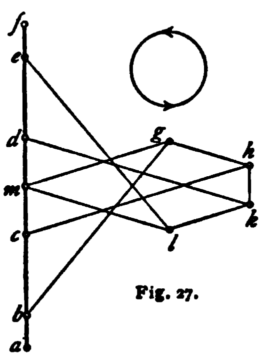


Fig. 27.

Considering now the polygon *chgmabc* in Fig. 26*a*, the forces represented by its sides are in equilibrium since the polygon is closed. Referring to Fig. 25 it is seen that the stresses in *CH*, *HG*, and *GM* are in equilibrium with the reaction *MA* and loads *AB* and *BC* on the left of a section cutting these members. But *chgmfedc* is also a closed polygon, hence the stresses in the same members are in equilibrium with the reaction and loads on the right of the section. Thus is again demonstrated the principle that the internal stresses in any section hold in equilibrium all the external forces on either side of the section (Art. 7).

The stress diagram for snow load, Fig. 27, is next constructed in a similar manner to that for dead load. As the snow loads are here laid off in the reverse order from the dead loads, the diagram is situated on the right of the load line instead of on the left as in the preceding figure.

The lines in all the diagrams are to be drawn with a well-sharpened pencil pressed lightly on the paper so as to produce a very fine line. As soon as an intersection is obtained it is to be marked with a needle point, enclosed with a small circle, and designated by the proper letter; other lines drawn to or from that point are not to pass within the circumference of this circle. The triangle and straight edge used in drawing parallel lines should be so arranged as to require the triangle to be moved the shortest distance. Special care is needed to hold the pencil at the same inclination from the beginning to the end of a line, or the line will not be strictly parallel to the edge of the triangle.

The following results, expressed in pounds, are now obtained by applying the scale of force to the stress diagrams:

TRUSS MEMBERS.	$BG = EL$	$CH = DK$	$GM = LM$	$GH = KL$	HK
Dead load stresses.	− 3 860	− 3 860	+ 2 580	+ 1 280	+ 750
Snow load stresses.	− 3 500	− 3 640	+ 2 340	+ 1 310	+ 600

When but a slight difference is found between the lengths of any pair of symmetrical lines in the stress diagram the average of the two is to be used; otherwise the diagram should be re-drawn.

As a final check the stresses in *CH* are computed (Roofs and Bridges, Part I, Art. 5), the lever arm of *CH* being measured on the truss diagram and found to be 7.27 feet. The stress due to dead load thus obtained is 3 861 pounds, and that due to snow load is 3 640 pounds.

Prob. 25. The upper chord of a wrought iron truss like Fig. 24 is divided into six equal panels, the triangle formed by the middle panel of each main rafter with the adjacent braces being isosceles. The span is 78 feet, the rise 19 feet 6 inches, and the trusses are 16 feet 6 inches apart, center to center. Find the dead and snow load stresses.

ART. 18. A TRIANGULAR ROOF TRUSS.

In the wrought iron truss in Fig. 28 the upper chord is divided into eight equal panels, verticals are drawn through the apexes, and the diagonals slope upward toward the center. Let the span be 80 feet, the rise of peak 15 feet, the lower

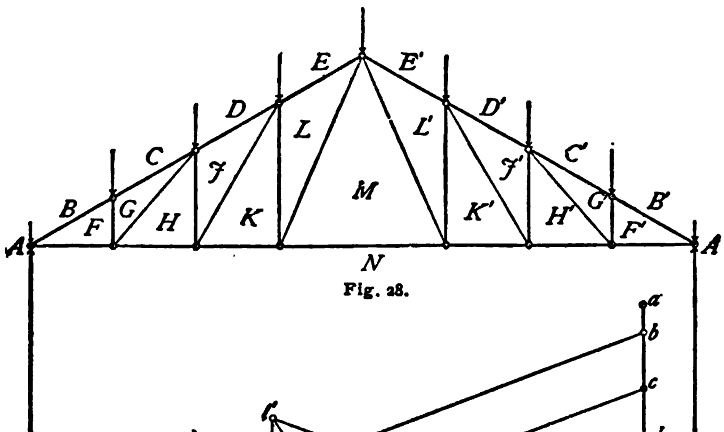


Fig. 28.

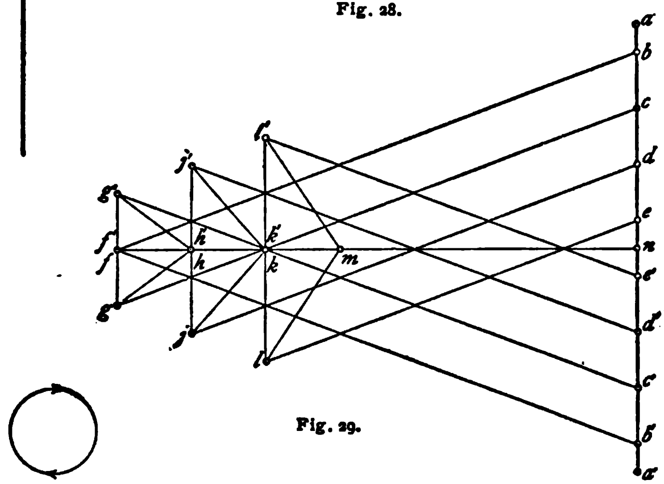


Fig. 29.

chord horizontal, and the distance between adjacent trusses 18 feet, center to center.

The main rafter is found to be 42.72 feet, and each panel 10.68 feet long. The weight of the truss is 9 720 pounds, and of the roof covering 18 455 pounds, making the total dead load

28 175 pounds. The dead apex load is one-eighth of this amount, or 3 522 pounds, and the half apex loads at the supports are each 1 761 pounds.

Since the inclination of each panel of the upper chord is the same the snow apex loads are equal, each one being $10 \times 18 \times 15 = 2\,700$ pounds. The ratio between any snow apex load and a dead apex load being $\frac{2\,700}{3\,522} = 0.7666$, the stresses caused by the snow load will bear the same ratio to those due to dead load, and hence only the dead load stress diagram is required.

The construction is begun by laying off the load line aa' equal to 28 175 pounds by scale and bisecting it at n. The half apex loads ab and $a'b'$ are next laid off equal to 1 761 pounds and bb' divided into seven equal parts. The reactions are $a'n$ and na, closing the polygon of external forces. The polygons representing the forces acting at each joint are successively formed, as explained in Art. 17, by beginning at the left support, passing to joints alternately on the upper and lower chords until the peak is reached, then going to the right support and passing from joint to joint until the peak is reached again. The last line to be drawn is $l'm$, which must be parallel to $L'M$ and pass through the intersection m of the lines lm and nm. The diagram if accurately drawn will be symmetrical with respect to nf; the distances fh, hk, and km will be equal, the points g, j, and l lying on a line parallel to $f'b'$ drawn through a point below b' on the load line at a distance equal to $b'c'$. Again, cg and $c'g'$ will pass through the intersection k of nf and ll'. As a final check the stress in MN may be computed; thus,

$$(14\,088 - 1\,761)40 - 3\,522(30 + 20 + 10) - S \times 15 = 0;$$

from which the value of S is $+ 18\,784$ pounds.

The scale of this stress diagram should be such that the line bf will not be longer than the main rafter of the truss diagram. The dead load stresses in the following table were obtained by using scales of 5 feet to an inch and 4000 pounds to an inch. The stresses due to snow load are then found by multiplying the dead load stresses by the ratio 0.7666.

Truss Members.		Dead Load Stresses.	Snow Load Stresses.
		Pounds.	Pounds.
Upper chord,	*BF*	− 35 090	− 26 900
	CG	− 35 090	− 26 900
	DJ	− 30 080	− 23 060
	EL	− 25 070	− 19 220
Lower chord,	*FN*	+ 32 870	+ 25 200
	HN	+ 28 180	+ 21 600
	KN	+ 23 480	+ 18 000
	MN	+ 18 780	+ 14 400
Braces,	*FG*	− 3 520	− 2 700
	GH	+ 5 870	+ 4 500
	HJ	− 5 280	− 4 050
	JK	+ 7 060	+ 5 410
	KL	− 7 040	− 5 400
	LM	+ 8 450	+ 6 480

Prob. 26. A wooden truss of the type of Fig. 28 has a span of 60 feet, rise 15 feet, and distance between trusses 14 feet. The upper chord is divided into six equal panels. Find the apex loads, reactions, and the dead and snow load stresses in all the members.

ART. 19. WIND LOADS.

The pressure produced by the wind on a roof surface depends on the direction and velocity of the wind and on the inclination of the roof. The wind is supposed to move horizontally, and a hurricane at 100 miles per hour exerts a pressure of probably 50 pounds per square foot of surface normal to its direction.

In determining the stresses due to wind it is often specified that the wind pressure shall be taken at 40 pounds per square foot of vertical surface.

While the subject is not fully understood, experiments show that the resultant pressure of a horizontal wind on an inclined surface may be represented by a normal force varying with the roof inclination. The following values deduced from experiments give the normal pressure per square foot for a horizontal wind pressure of 40 pounds per square foot for different inclinations of the roof surface:

INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.
5°	5.1	25°	22.6	45°	36.0
10°	9.6	30°	26.5	50°	38.1
15°	14.2	35°	30.1	55°	39.4
20°	18.4	40°	33.3	60°	40.0

For all inclinations exceeding 60 degrees the normal pressure is 40 pounds per square foot, and for intermediate inclinations the pressures are obtained by interpolation. Should the horizontal wind pressure be assumed lower or higher than 40 pounds the normal pressure is to be changed in the same ratio.

Let it be required to find the wind apex loads for the truss whose dimensions are given in Fig. 30. The inclination of AB is found to be 50° 40′ and that of BC 16° 40′, hence from the above table the normal wind pressures are respectively 38.3 and 15.6 pounds per square foot. If the trusses be 12 feet apart the total normal wind pressure on AB is

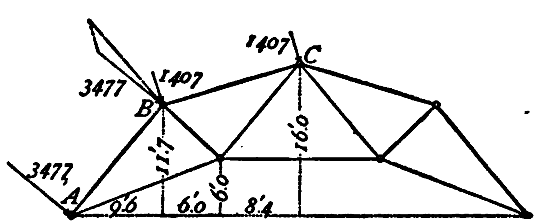


Fig. 30.

$$\sqrt{9.6^2 + 11.7^2} \times 12 \times 38.3 = 6\,954 \text{ pounds,}$$

one-half of which is applied at A and one-half at B, as shown. In the same way the wind upon BC brings at B and C two normal apex loads, each of 1 407 pounds.

The two apex loads at B are then combined by means of the force triangle, the resultant being 4 711 pounds.

Prob. 27. Find the wind apex loads for the truss in Fig. 25, using the dimensions given in Art. 17.

ART. 20. A TRUSS WITH FIXED ENDS.

Roof trusses of short span, especially wooden trusses, generally have both ends firmly 'fixed' to the supporting walls. The reactions caused by the wind pressure are inclined and their horizontal components tend to overturn the walls of the building.

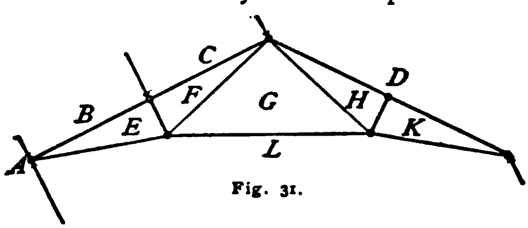


Fig. 31.

Let the truss in Fig. 31 have both ends fixed, the span being 40 feet, the rise of peak 10 feet, the rise of horizontal tie rod 2 feet, and the distance between trusses 12 feet.

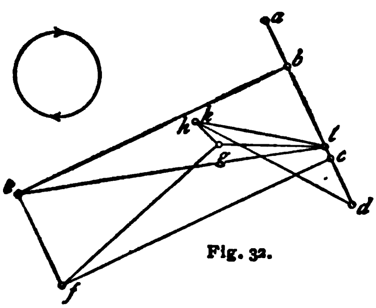


Fig. 32.

The length of the main rafter is found to be 22.36 feet, its inclination 26° 34′ and the normal wind pressure (Art. 19) 23.8 pounds per square foot of roof surface. The apex load BC is

$$\tfrac{1}{2} \times 22.36 \times 12 \times 23.8 = 3\,193 \text{ pounds} = 3.2 \text{ kips},$$

the word 'kip' meaning 1 000 pounds.

As the sum of all the external forces in the direction of the wind loads must equal zero, the reactions are parallel to them.

The lines of action of the reactions at the supports being therefore known their magnitudes are determined by the method of Art. 8. The results are 4.4 kips for the left reaction, and 2.0 kips for the right reaction. These may now be checked by computation. A perpendicular from the right support to the left reaction measures 35.78 feet. The loads *AB* and *CD* are each one-half of the load *BC*. Then taking moments about the right support,

$$AL \times 35.78 - 1.6 \times 35.78 - 3.2 \times 24.6 - 1.6 \times 13.42 = 0,$$

whence $AL = 4.4$ kips. In a similar manner, by taking moments about the left support, the reaction *DL* is found to be 2.0 kips. The same result is obtained both graphically and analytically by replacing the apex wind loads by their resultant, 6.4 kips, applied at the middle of the rafter.

The stress diagram is begun by drawing *ad* normal to the main rafter and equal to 6.4 kips by the scale of force, while *ab* and *cd* are each made one-fourth of the length of *ad*. The diagram is then completed in the manner described in preceding articles. As *h* and *k* are found to coincide it shows that there is no stress in *HK* when the wind blows on the left side of the truss.

MEMBERS.	STRESSES.
	Kips.
BE	− 9.36
CF	− 9.36
EL	+ 9.76
GL	+ 3.36
EF	− 3.20
FG	+ 6.64
DK	− 5.58
DH	− 5.58
LK	+ 4.16
HK	0
GH	+ 1.04

With scales of 3 feet to an inch and 1 ton to an inch the results given in the table were obtained. As a check the stress in *GL* is computed, the same value being found as that given in the table.

If the wind loads be placed on the right-hand side of the truss the corresponding stress diagram will be the same as Fig. 32 when revolved about a vertical axis, therefore only one stress diagram is required in this case. Accordingly the stresses in the members of either half of the truss when the wind blows on the right are the same as the stresses in the corresponding members of the other half of the truss for wind on the left side. For instance, the stress in *GH* is + 6.64 kips and in *EF* the stress is zero for wind on the right.

Prob. 28. Find the wind stresses in all the members of the truss in Prob. 26, both ends being firmly bolted to the walls.

Art. 21. A Truss with One End Free.

Changes in temperature cause expansion and contraction in iron trusses which if both ends are fixed give rise to certain stresses. To avoid these only one end is fastened to the supporting wall, the other being merely supported or 'free,' so that it may move horizontally in the direction of the length of the truss.

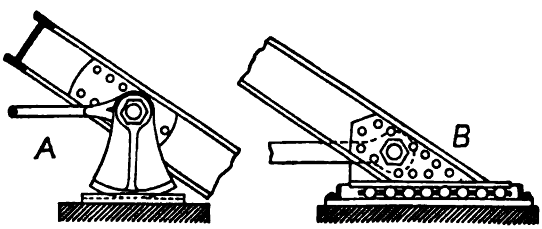


Fig. 33.

The free end may rest upon a smooth iron plate upon which it slides, but this arrangement requires too much friction to be overcome in the case of heavy roofs, especially if the plate becomes rusty. Sometimes it is attached to a rocker, as at *A* in Fig. 33, and often rollers are employed, as shown at *B*. If no friction exists at the free end the reaction there is vertical.

In determining the stresses due to wind when one end is fixed and the other free it is necessary to construct two diagrams, one for the wind blowing on the fixed side and the other for the wind load on the free side.

For example, let the truss in Fig. 34 be taken, the span being 76 feet, rise of upper chord 19 feet, rise of lower chord 4 feet, and the distance between trusses 15 feet. The web members consist of verticals and diagonals as shown, the chords being divided into eight equal parts.

The inclination of the upper chord is the same as that in the last article, hence the normal wind pressure per square foot

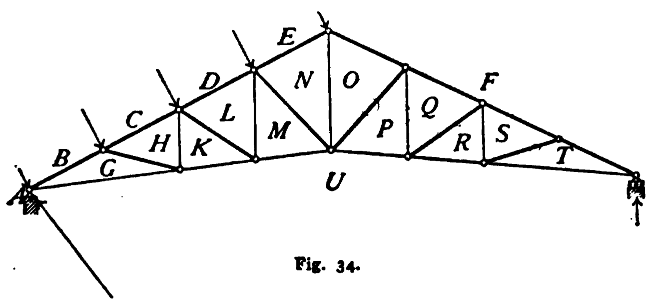


Fig. 34.

is 23.8 pounds, the apex loads *BC*, *CD* and *DE* are 3.8 kips each and the loads *AB* and *EF* are 1.9 kips each.

The reaction at the free end being vertical, its value is most readily found by computation. Taking moments about the left support and considering the total wind load concentrated at the middle of the rafter,

$$15.2 \times 21.24 - FU \times 76 = 0, \text{ whence } FU = 4.24 \text{ kips.}$$

In Fig. 35 let *af* be drawn perpendicular to the loaded upper chord and made equal to 15.2 kips; *ab* and *ef* are then laid off equal to 1.9 kips each, and *be* divided into three equal parts. The vertical reaction *u* is next drawn equal to 4.24 kips, and as the other reaction must close the force polygon, *ua* repre-

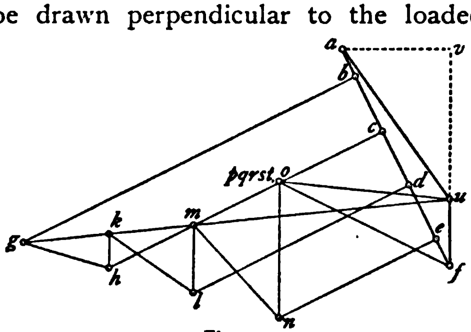


Fig. 35.

sents the magnitude and direction of the reaction *UA* at the fixed end.

The student should notice particularly that it is not possible for the reaction at the fixed end to be parallel to the wind loads when one end rests on rollers.

The stress diagram is constructed in the usual manner by beginning with the forces acting at the left support, and passing to joints alternately on the upper and lower chords. After the joint *DENML* is reached the forces at the peak are taken instead of passing to the central joint of the lower chord. The force polygon for the peak is *nefon*, the point *o* being determined by it. Now passing to the joint below the peak the corresponding force polygon requires a parallel to *UP* to be drawn through *u*, and a parallel to *OP* through *o*. It is found however that the former parallel passes through *o*, thus closing the polygon and reducing *op* to zero. But *uo*, *of*, and *fu* form a closed force triangle which indicates that *O* belongs to the entire space between the chords in the right half of the truss; therefore in this part of the truss each chord has the same stress in every panel and no web member is stressed when the wind blows on the opposite side.

The same thing may be shown in a different manner by beginning at the right support where the reaction *FU* is held in equilibrium by the stresses in *UT* and *FT*. The force triangle *fuo* represents this relation, *uo* being the stress in *UT* and *fo* that in *FT*. Passing to the next joint on the upper chord it is required to draw a triangle two of whose sides shall be parallel to the straight upper chord and the third side parallel to *ST*. This causes the two sides to coincide and the third side to disappear, hence the stress in *FS* equals that in *FT* and the stress in *ST* is zero. The same conditions occur at each joint on the upper and lower chords to the right of the middle of the truss.

If the stress diagram be accurately drawn, the point *m* marks the intersection of *ch*, *ug*, *lm*, and *nm*. The points *g*, *h*, *l*, and *n* will lie in a straight line and be equidistant.

When the wind blows upon the free side of the roof, as in Fig. 36, the apex loads are the same as before, and the reaction

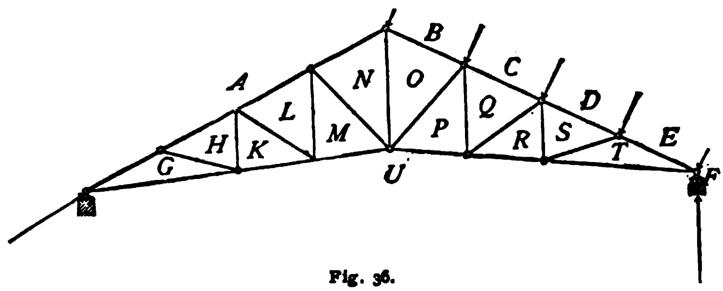


Fig. 36.

FU equals 9.36 kips. After *fa* in Fig. 37 is laid off, *fu* is drawn vertically equal to 9.36 kips by scale, its length being the same as *uv* in Fig. 35. In this stress diagram which is completed similarly to Fig. 35, *an* represents the stress in the upper chord, and *un* that in the lower chord, of the left half of the truss. The braces on the left of *NO* are not affected by the wind on the right.

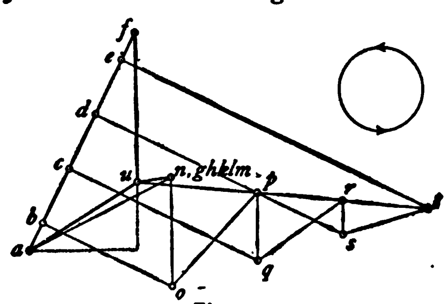


Fig. 37.

The actual construction of the diagrams for this example was made to scales of 6 feet to an inch and 4 kips to an inch, and the results are shown in Figs. 38 and 39. The stresses are marked on the skeleton truss diagrams for convenient comparison, the members in compression being indicated by heavy lines and the tensile members by light lines. The checks by computation give -12.02 kips for the stress in *FO*, Fig. 34, and -10.00 kips in *AN*, Fig. 36.

It is seen that greater stresses are produced in the chords except *EN*, and in the center vertical when the wind blows on

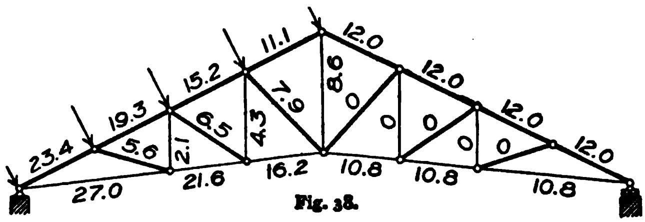


Fig. 38.

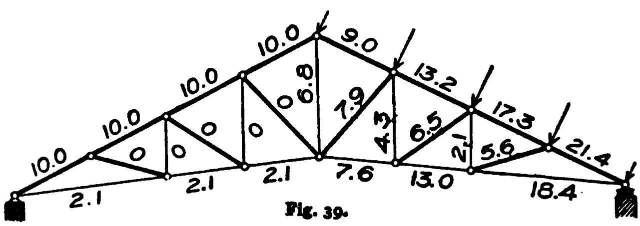


Fig. 39.

the fixed side, while the stresses in the braces to the windward with the exception of *NO*, are the same for the wind blowing on either side.

The reactions may also be obtained graphically. The direction of the reaction at the fixed end is unknown, but its line of action passes through the joint at that end, hence the equilibrium polygon should be drawn from that joint to a vertical through the support at the free end. The ray drawn parallel to the closing line will then intersect the vertical through *f* at the point *u*.

Prob. 29. Determine the reactions for the example in this article by the equilibrium polygon.

Prob. 30. Find the wind stresses for the truss in Fig. 28, using the dimensions given in Art. 18. The right end is to rest on rollers.

ART. 22. ABBREVIATED METHODS FOR WIND STRESS.

When the lower chord is horizontal the stresses in the main rafter either to the windward or to the leeward are the same for the wind blowing either on the fixed or on the free side. The difference between the stresses in the horizontal chord

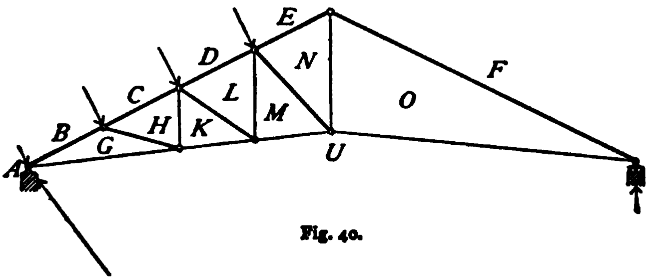


Fig. 40.

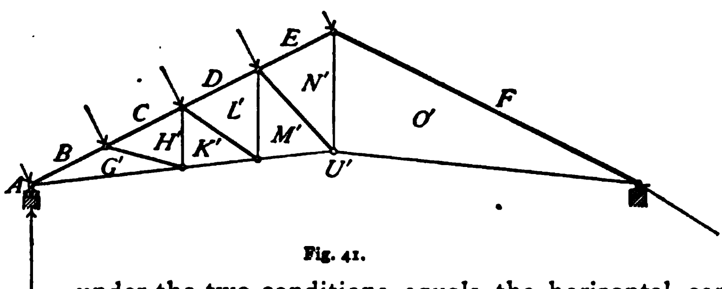


Fig. 41.

under the two conditions equals the horizontal component (*av*, in Fig. 35) of the wind loads, the tension being less when the wind blows on the free side. In such a case, then, only one wind stress diagram is really needed.

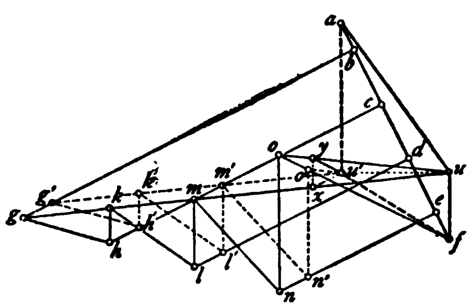


Fig. 42.

Even if the lower chord is raised, a few additional lines on the diagram for wind on the fixed side render the other diagram unnecessary. The full lines in Fig. 42 are simply a copy of Fig. 35 and the truss diagram in Fig. 40 is lettered to correspond, the rollers being at the right support. Now supposing the wind loads to remain unchanged and the rollers to be transferred to the left support, the corresponding stress diagram is that shown in broken lines. This relation of the wind loads, the truss, and the rollers is illustrated in Fig. 41.

Let a horizontal line be drawn through u until its meets the vertical through a at u', and join u' with f and a thus giving the new reactions fu' and $u'a$. Let $u'x$ be drawn parallel to bg meeting ug at x, and $u'y$ parallel to fo meeting uo at y. The line joining x and y will be parallel to no.

The stresses in the main rafter are changed when the rollers are transferred to the left support by the amount $gg' = hh' = ll' = nn' = u'x = u'y = oo'$, those in the lower chord are changed by $ux = uy$, and that in NO by xy, while the remaining stresses are unaltered.

Applying the scale, $u'x$ and $u'y$ are each found to be 2.04 kips, the differences ux and uy are each 8.66 kips and xy measures 1.80 kips. In the following table the first line contains the stresses obtained from Fig. 35, and after subtracting the changes of stress, the same results are obtained as those derived from Fig. 37:

STRESSES FOR WIND ON THE LEFT.

TRUSS MEMBERS, .	*BG*	*CH*	*DL*	*EN*	*FO*	*UG*	*UK*	*UM*	*UO*	*NO*
Rollers on right, . .	23.4	19.3	15.2	11.1	12.0	27.0	21.6	16.2	10.8	8.6
Changes in stresses, .	2.0	2.0	2.0	2.0	2.0	8.7	8.7	8.7	8.7	1.8
Rollers on left. . .	21.4	17.3	13.2	9.0	10.0	18.3	12.9	7.5	2.1	6.8
TRUSS MEMBERS, .	*BG'*	*CH'*	*DL'*	*EN'*	*FO'*	*U G*	*U'K'*	*U M*	*U O'*	*N'O'*

For any other type of truss the changes are likewise easily obtained. When the middle panel has a horizontal tie as in the example given on Plate I, the form of the auxiliary polygon $u'xzy$ is somewhat different from the preceding one and the change for the horizontal tie is measured from k to z. The following measurements were obtained from the original diagram which was made with a scale of 4 kips to an inch : $u'x=$

$u'y = 1.76$ kips, $kx = ky = 8.58$ kips, $kz = 8.34$ kips, and $xz = yz = 0.80$ kip.

Prob. 31. Prepare a table of wind stresses similar to the above for the example given on Plate I.

Prob. 32. Find the wind stresses for a wooden truss like Fig. 34 whose span is 74 feet, rise of peak 19 feet, the lower chord being horizontal and the trusses 16 feet 3 inches apart.

ART. 23. COMPLETE STRESSES FOR A TRIANGULAR TRUSS.

On Plate I are given the dimensions of a wrought iron roof truss together with the specified loads. The struts are normal to the rafters as shown on the skeleton outline of the truss. All the diagrams required to determine the stresses due to dead, snow, and wind loads are shown, and are constructed in the manner explained in the preceding articles of this Chapter. The stresses as measured by scale (2 tons to an inch on the original) are arranged in tabular form.

The preliminary computations give the following results:

Length of rafter, ...	41.15 feet.
Weight of truss, ...	7.70 kips.
Weight of roof covering, ...	16.30 kips.
Total dead load, ...	24.00 kips.
Dead apex load, ...	3.00 kips.
Dead load reaction, ...	12.00 kips.
Snow apex load, ...	2.28 kips.
Ratio of snow load stresses to dead load stresses, .	0.76.
Inclination of roof surface, ...	25° 57′.
Normal wind pressure per square foot of roof,	23.3 pounds.
Total wind load, ...	15.84 kips.
Wind apex load, ...	3.96 kips.
Horizontal component of total wind load, . . .	6.94 kips.
Vertical component of total wind load, ...	14.24 kips.
Reaction at free end for wind on fixed side, . .	4.40 kips.
Reaction at free end for wind on free side, . .	9.84 kips.

The reactions are obtained graphically as follows: Supposing both ends of the truss to be fixed, the reactions due to wind on the left side are *au* and *ua* in the stress diagram marked 'wind on fixed side.' They are obtained by the method of Art. 8. The pole is at *o* and the equilibrium polygon is *rst*, the wind apex loads being concentrated at apex 2. But since the right end of the truss rests on rollers the reaction *AK* of the right support is vertical and is represented by the line *ak* or the vertical component of *au*. The closing side *ka* of the force polygon represents the reaction of the left support. Applying the scale the value of *ak* is found to be 4.4 kips, which is also the value of the vertical component of the reaction of the left support when the wind blows on the right side.

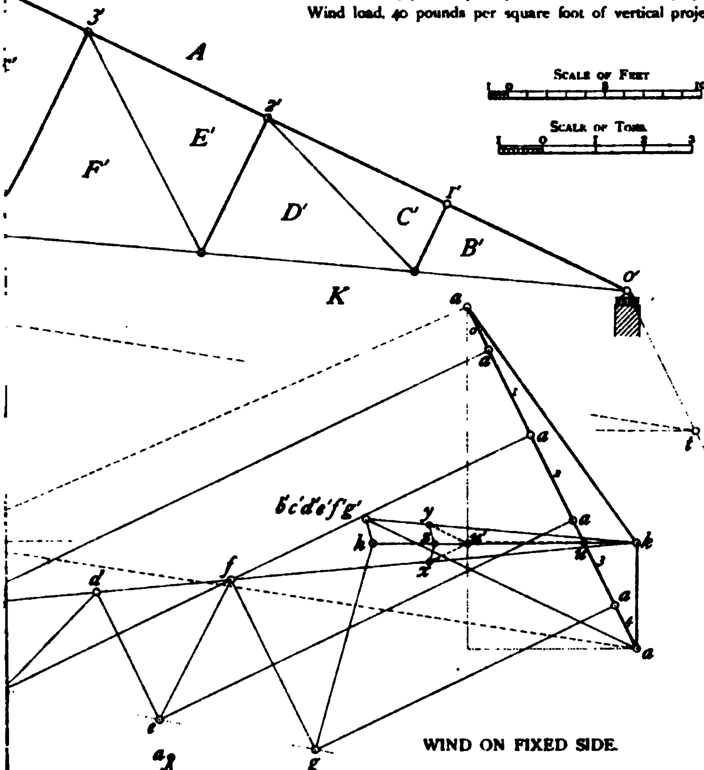
In order to design a member for the range of stress (Mechanics of Materials, Art. 137), it is necessary to know the minimum stress as well as the maximum stress to which it is subjected by the combined loads. As the dead load always acts its effect must be included in finding both the minimum and the maximum stresses. Snow load always produces stresses of the same kind as the dead load when the rafters are straight, and hence is used only in obtaining the maximum. As the wind cannot blow on more than one side of the roof at the same time, only one of the wind stresses is to be combined with the dead, or with the dead and snow load stresses. In the present example, all the stresses in any member are either tensile or compressive, hence the minimum equals the dead load stress and the maximum equals the sum of the dead, snow, and larger wind load stresses. The stresses in the table on Plate I are given in net tons, and these may be transformed into kips, or thousands of pounds, by multiplying by 2. The student should make the table in kips for his given data.

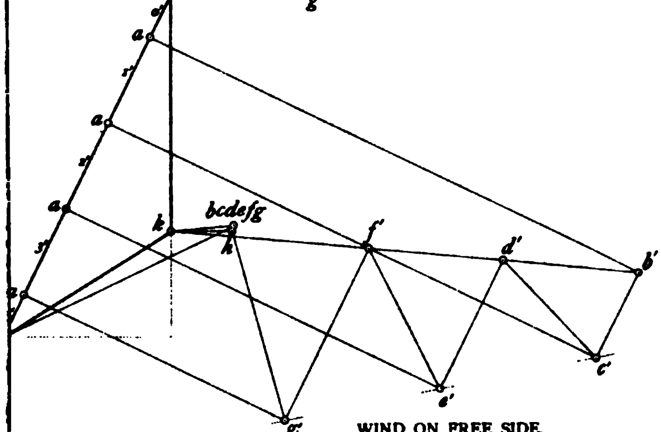
It is seen from the table that the maximum chord stresses are greater on the fixed side than on the free side, while the maximum stresses in the bracing are the same on both sides.

:F TRUSS.

LOADS.

Weight of wrought iron truss, see formula.
Roof covering, 12 pounds per square foot of roof surface.
Snow load, 15 pounds per square foot of horizontal projection.
Wind load, 40 pounds per square foot of vertical projection.





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Prob. 33. A wrought iron truss of the type shown in Fig. 28 has a span of 76 feet, rise of peak 18 feet, and rise of tie *MN* 3 feet. The trusses are 16 feet 6 inches apart, their right ends resting on rollers. Find the maximum and minimum stresses in all the members.

ART. 24. COMPLETE STRESSES FOR A CRESCENT TRUSS.

In the crescent truss whose outline and general dimensions are given on Plate II the joints on both the upper and lower chords lie on arcs of circles, and the alternate braces are radials of the upper circular arc.

The following dimensions and apex loads are obtained either graphically or by computation, and in some cases by both methods, one being a check on the other:

Radius of circle containing the joints of the upper chord, ... 51.25 feet.
Radius of circle containing the joints of the lower chord, ... 99.06 feet.
Length of panels on the upper chord, ... 11.08 feet.
Weight of truss, ... 8.24 kips.
Weight of roof covering, ... 17.04 kips.
Total dead load, ... 25.28 kips.
Dead apex load, ... 3.16 kips.
Dead load reaction, ... 12.64 kips.
Horizontal projection of panels of upper chord,
8.04, 9.44, 10.53, 10.99 feet.
Snow apex loads, . . . 0.96, 2.10, 2.40, 2.58, 2.64 kips.
Snow load reaction, ... 9.36 kips.
Inclinations of roof surface, . 43° 20′, 31° 05′ 18° 30′, 6° 10′.
Normal wind pressures per sq. ft., 35.2, 27.3, 17.2, 6.2 pounds.
Total wind loads on the panels, . 6.24. 4.88, 3.04, 1.08 kips.

The dead load stress diagram is constructed in the same way as in previous examples, and is symmetrical with respect to a horizontal axis through *k*. As the snow apex loads are not uniform a separate diagram for stresses due to snow loads is required, and this one is also symmetrical with respect to a horizontal axis through *k*.

The wind apex loads are next obtained by combining half of the wind loads on the panels adjacent to each apex as illustrated in Art. 19. On the truss diagram in Plate II the wind apex loads are drawn to double the scale of tons in order to determine their directions with greater precision. The reactions are then obtained by means of the equilibrium polygon (Arts. 6, 8, and 21). The ray *ou'*, parallel to the closing side of the polygon, intersects the resultant of the wind loads *au'a* at the point *u'* giving *au'* and *u'a* as the reactions of the right and left supports if both ends of the truss were fixed, but as the right end is free its reaction *ak* is vertical and equal to the vertical component of *au'*. The load line for wind on the free side is obtained from that for wind on the fixed side by revolving it about a vertical axis, which operation may be conveniently performed by means of a piece of tracing paper. The point *u''* in this diagram is the same as *u'* in the preceding one, hence only one equilibrium polygon is required. The reaction *ka* of the right support is the vertical component of *u''a*.

It is observed that when the wind blows on the free side of the truss, it causes compression in the lower chord from *KH'* to the left end. The stresses in the upper chord are also considerably less than for wind on the fixed side, and if the rise were a little greater it would cause tension in one or more panels near the left support.

The truss diagram for this example should be twice as large as that on Plate II so that no line in any stress diagram would be longer than the truss member to which it is parallel. This relation was here disregarded as the limited size of the plate would have reduced the stress diagrams to indistinctness.

Unless special care is exercised in drawing the wind stress

CR

G

K/r
+ 10.1
+ 7.7
+ 2.7
+ 4.8
+ 22.6
+ 10.1

diagrams they will not close. In all cases the work should proceed from each support toward the center of the truss.

Upon the completion of the stress diagrams their lines are measured by the scale of force, the stresses arranged in a table and the maximum and minimum stresses found by taking the algebraic sum of dead, snow and larger wind stress for each member. For instance, the maximum stress in *KD* is + 21.6 + 17.0 + 19.4 = + 58.0 kips, and its minimum stress is + 21.6 − 7.2 = + 14.4 kips. The maximum stress in *GH* is − 0.2 + 4.6 = + 4.4 kips and the minimum stress in the same member is − 0.2 − 0.2 − 3.4 = − 3.8 kips. In the table of Plate II stresses are given in net tons, one ton being two kips, but the student should preferably use kips.

Prob. 34. Find the maximum and minimum stresses in all the members of Fig. 30, the right end resting on rollers.

ART. 25. AMBIGUOUS CASES.

When, in the determination of the stresses in the Fink truss shown in Fig. 43, the joint at the middle of the rafter is reached the load *BC* and the stresses in *BG* and *GH* are known, leaving three stresses unknown, namely, those in *CL*, *LK*, and *KH*. As the resultant of the load and the known stresses cannot be resolved into more than two given directions, another condition needs to be added.

If the loads *AB* and *CD* be equal, as in this example, the symmetrical relation of *GH* and *LK* causes them to have equal stresses and therefore *fg* and *lm* lie in the same straight line parallel to *hk*. The polygon *hgbclkh* is then readily completed. If the loads *AB* and *CD* be unequal, the panels remaining equal on the upper chord, the polygon may be drawn by noting that the point *k* must lie midway between the parallels *cl* and *dm*. This follows from the fact that *LM* is normal to the upper chord and that KL and MN are of equal length and make the same angle with LM as well as with the upper chord. The triangle lkm is hence an isosceles triangle.

If CL and DM be of unequal lengths then both of these methods fail. A general solution of this problem was given by

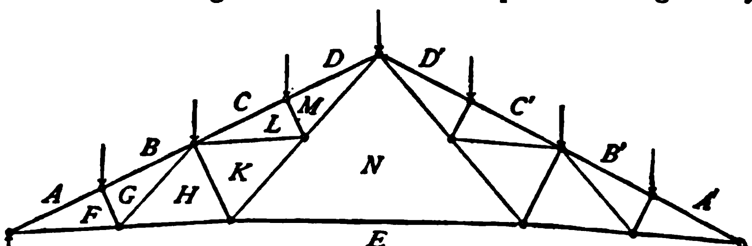


Fig. 43.

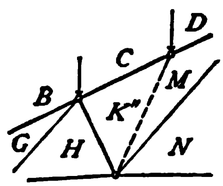


Fig. 44.

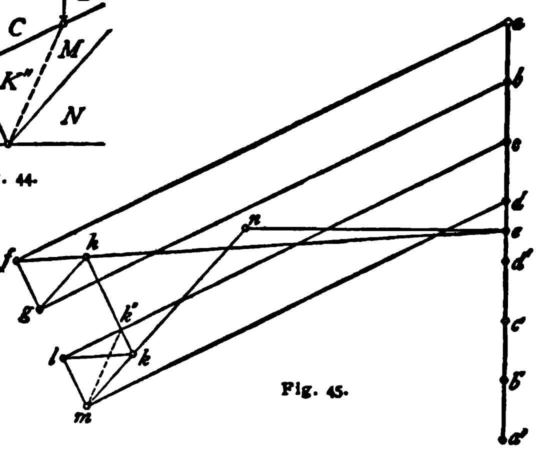


Fig. 45.

WILLETT in a paper read before the Chicago Chapter of the American Institute of Architects, March 22, 1888, which consists in temporarily changing the webbing of the truss.

Let the braces KL and LM be removed and the diagonal $K''M$ be substituted as shown in Fig. 44. The load BC and the stresses in BG and GH being known, those in HK'' and $K''C$ are found from the polygon $hgbck''h$ in Fig. 45. For the next apex on the upper chord the polygon is $k''cdmk''$, the line mk'' being the unknown stress thus determined. Passing to the joint $HK''MNE$ where three stresses are now known, the polygon $ehk''mne$ gives the unknown stresses mn and ne. As the line mn is now fixed, the original webbing is restored and the remaining parts of the stress diagram drawn; hk'' is produced to meet mn at k, and kl and ml are drawn parallel to KL and ML respectively to meet ck'' produced at l.

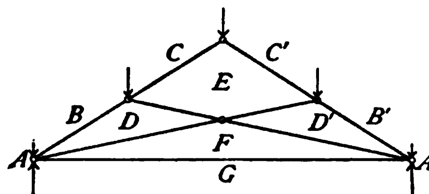


Fig. 46.

The remaining half of the diagram is not shown in Fig. 45. If each half of the truss and the loading upon it be equal to the other, the complete stress diagram will be symmetrical with respect to a horizontal axis through e.

In the truss whose outline is given in Fig. 46 it is not possible to begin the stress diagram by considering the forces acting at the left support, since three unknown stresses hold in equilibrium the known reaction and half apex load.

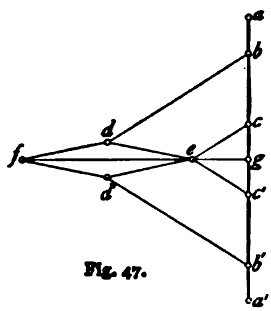


Fig. 47.

At the peak the load CC' is supported by two members whose stresses are $c'e$ and ec in Fig. 47. The quadrilateral $bced$ gives the stresses in the members meeting at the joint $BCED$, and $c'b'd'e$ gives those for the corresponding joint in the right half of the truss. Passing to the joint below the peak the stress polygon is found to be $ded'f$. For the left support the reaction, half apex load, and the stresses bd and df are known, but one stress remains unknown. As equilibrium exists at this joint the polygon must be closed by the line fg, which is also to be parallel to FG. This completes the diagram, and by following around the polygons all the stresses are found to be compression except *fg*, which is tension.

Suppose the tie *FG* to be omitted. Each reaction must then be inclined in order to maintain equilibrium, its horizontal component being equal to *fg*. The reaction of the right support is therefore *a'f* (not drawn) and that of the left support *fa*.

Prob. 35. Find the dead load stresses in the wooden truss of Fig. 48, the span being 36 feet, the rise 12 feet, and the distance between trusses 12 feet. Then find the stresses for a truss like Fig. 46 of the same dimensions and compare the results.

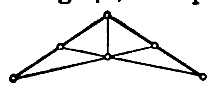


Fig. 48.

Prob. 36. A wooden truss like Fig. 49 has a span of 48 feet and a rise of 12 feet, both ends being fixed. Find the maximum and minimum stresses, the trusses being 14 feet apart.

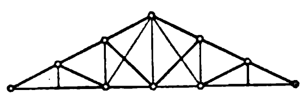


Fig. 49.

ART. 26. UNSYMMETRICAL LOADS AND TRUSSES.

In mills and shops, loads are frequently suspended from the lower chords of the roof trusses, as for instance, lines of shafting, etc. In such cases it is convenient to determine the stresses due to the loads by means of a separate diagram, as illustrated in Fig. 50. The load *AB* produces no stress in *DE* or *GH*, while it causes stresses in the other members of the same kind as those due to dead, snow, and wind loads, and hence their maximum stresses are increased. In special cases such sus-

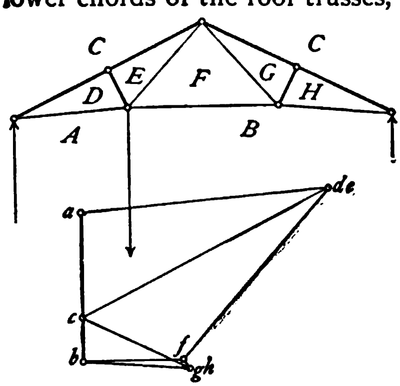


Fig. 50.

pended loads may even change some maximum stresses due to the other loads from compression to tension or from tension to compression.

When a ceiling is attached to the lower chord it becomes a part of the dead load and needs no separate diagram. It is combined at once with the other loads in the dead load diagram, all the loads and reactions being taken in regular order around the truss and laid off on the load line, some portions of which will be found to overlap.

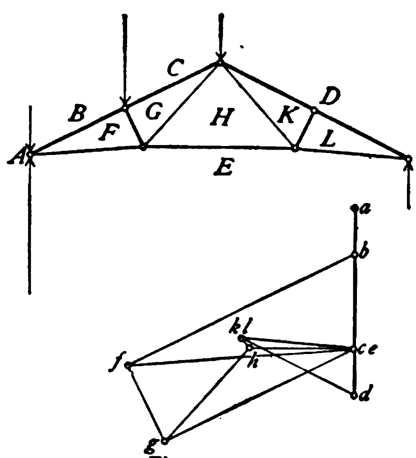


Fig. 51.

It was formerly the practice in England to find the effect of the wind on roof trusses by taking vertical loads of 20 pounds or more per square foot of horizontal projection, acting upon one side of the roof. Fig. 51 shows a truss under such loads and the resulting stress diagram. It is observed that the stress thus caused in *BF* is greater than that in *CG*, while under normal wind loads they are equal.

In Fig. 52 is given an unsymmetrical truss under the action of dead load. The stress diagram is hence unsymmetrical and has also fewer checks upon its construction. The main check however still remains, which requires that after working from each support toward the peak the closing line *op* shall be parallel to the member *OP*. To determine the stresses in an unsymmetrical truss by the analytic method materially increases the labor of computation required for a symmetrical truss, but with the graphic method it makes no difference whatever for any type of truss.

Prob. 37. Let the load AB in Fig. 50 be 5 000 pounds, and the dimensions of the truss the same as given in Art. 20. Find the stresses in all the members.

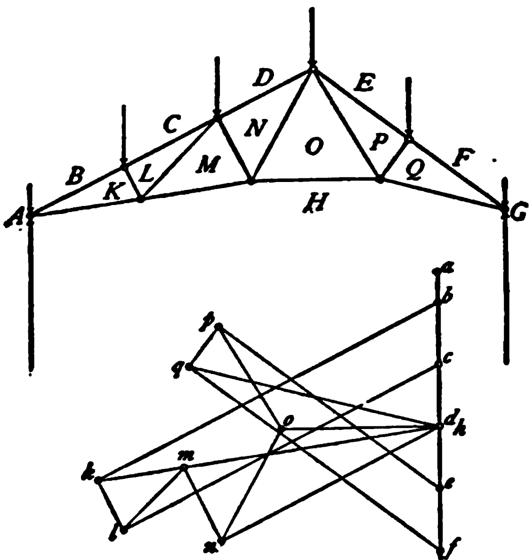


Fig. 52.

Prob. 38. Find the wind stresses for the same truss under a vertical wind pressure of 20 pounds per horizontal square foot, and compare the results with those obtained in Art. 20.

CHAPTER III.

BRIDGE TRUSSES.

ART. 27. LOADS ON BRIDGE TRUSSES.

The weight of the floor, lateral bracing, trusses, and all the pieces that unite and stiffen them, compose the dead load of a bridge. This weight depends upon the span, width, and style of the bridge, and upon the live load and unit stresses adopted in its design, and varies considerably in individual cases; it is usually lighter for a highway bridge than for a railroad bridge.

The total weight or dead load of a highway bridge with two trusses may be expressed approximately by the following empirical formula:

$$w = 140 + 12b + 0.2bl - 0.4l,$$

in which w is the weight in pounds per linear foot, b the width of the bridge in feet (including sidewalks, if any), and l the span in feet. The width of a highway bridge in the clear varies from 16 to 24 feet, which is only exceeded in large cities.

The following formulas for estimating the weights of bridges are taken from Design of Steel Bridges by F. C. KUNZ, published in 1915. For ordinary country highway bridges with riveted trusses, with spans not exceeding 200 feet, a width of roadway from 12 to 20 feet, and without sidewalks,

$$w = (0.12l + 12)(1.6 - 0.03b)b,$$

in which w is the weight in pounds per linear foot for the steel only. To this must be added the weight of the plank flooring. If sidewalks with steel joists are used, the weight of steel is to be increased by about 12 pounds per square foot of sidewalks. Through truss bridges with open-tie flooring, carrying exclusively electric cars weighing 30 tons each on a single track, have an approximate weight per linear foot for steel only, of $w = 2l + 200$. The following formulas give the weight of steel per linear foot for single track through railroad bridges with riveted or pin-connected trusses, and without end floorbeams, designed for COOPER'S E50 loading (see Art. 39), in accordance with the standards of the American Bridge Co. for 1902:

$$w = 7l + 640 \text{ for single track;}$$
$$w = 14l + 1200 \text{ for double track.}$$

To the weights thus obtained it is necessary to add from 350 to 450 pounds per linear foot for the weight of the ordinary open deck for each track. The weight of a ballasted deck is somewhat larger. When end floor beams are used, the total weight of the span is to be increased by 5 000 pounds for a single-track, and by 16 000 pounds for a double-track bridge. The corresponding formula for a single-track through truss bridge without end floor beams designed in accordance with the 1908 standards of the Pennsylvania Steel Co., with spans not exceeding 200 feet, is $w = 9l + 410$.

In a volume on Steel Railway Bridges by EDWARD C. DILWORTH, published in 1916, are given diagrams for the weights of simple-span truss bridges designed in accordance with the specifications of the American Railway Engineering Association, 1912, with drawings showing details, and tables giving weights of the steel; the weights are in accordance with the following formulas:

$w = (l + 40)/a$, for single-track deck bridges,
$w = (l + 45)/a$, for single-track through bridges,
$w = (l + 85)/a$, for double-track deck and through bridges,

in which w is the weight of steel in pounds per linear foot of span, including end floor beams and shoes, l the length of span in feet, and a a constant depending upon the type of truss bridge and loading. Values of a are given in the following table:

Live. Load	Deck Riveted Trusses. S.T.	D.T.	Through Riveted Trusses. S.T.	D.T.	Through Pin Trusses. S.T.	D.T.
E40	0.108	0.0701	0.115	0.0730	0.1265	0.0805
E50	0.094	0.0610	0.100	0.0635	0.1100	0.0711
E60	0.084	0.0545	0.089	0.0567	0.0981	0.0625

NOTE.—S. T. = Single Track, and D.T. = Double Track.

If struts are used instead of end floor beams, the following deductions are to be made from the total weight of a through span: Single-track riveted, 6000 pounds; double-track riveted, 15 000 pounds; single-track pin, 9 000 pounds; double-track pin, 16 000 pounds.

An elaborate set of diagrams for the weights of steel per linear foot of span is given in Chap. LV of Bridge Engineering by J. A. L. WADDELL, published in 1916. These weights are for simple truss bridges with single or double track, under WADDELL'S new standard live loads, with riveted or pin-connected trusses of the Pratt or Pennsylvania (Pettit) types. For each type of truss separate diagrams are given for the weights of steel in the floor system, the lateral systems, the trusses, and the metal on piers respectively, as well as for the total weight of metal per foot of span.

Other formulas relating to the dead load or weight of bridges may be found in Roofs and Bridges, Part I, Art. 20.

The live load is that which passes over the bridge, and consists of wagons, automobiles, motor trucks, and foot passengers on highway bridges, and of trains on railroad bridges.

The Massachusetts Public Service Commission in 1915 adopted the following live loads for highway bridge trusses which carry electric railways. For bridges intended for passenger cars the trusses shall be proportioned to carry on each track a train of two double-track cars coupled together. Each car shall be assumed to weigh, when loaded, 50 tons, and to have a total wheel base of 25 feet, and a wheel base for each truck of 5 feet. The length of each car shall be taken as 40 feet. For bridges over which it is intended to operate standard steam railroad freight cars, or express or other cars weighing more than 50 tons when loaded, the weight of each car shall be taken as 75 tons when loaded.

In addition to the electric cars the Commission specifies that the following moving loads are to be assumed upon the highway bridge floor: (*a*) For city bridges, subject to heavy loads: 100 pounds per square foot of floor surface for spans of 100 feet or less, and 80 pounds for spans of 200 feet or over, and proportionally for intermediate spans. This uniform load is to be taken as covering the floor up to within two feet of the rails. (*b*) For suburban or town bridges, or heavy country bridges: 80 pounds per square foot of floor surface for spans of 100 feet or less and 60 pounds per square foot for spans of 200 feet or over. (*c*) For light country highway bridges: 80 pounds per square foot of floor surface for spans 75 feet or less and 50 pounds for spans of 200 feet or more.

The live loads usually assumed for highway bridges which do not carry electric railways are as follows, in pounds per square foot of floor surface:

	For Country Bridges.	For City Bridges
Spans under 50 feet,	90	100
Spans 50 to 125 feet,	80	90
Spans 125 to 200 feet,	70	80
Spans over 200 feet,	60	70

This maximum load consists of a dense crowd of people covering the roadway and sidewalks, and as there is less liability to crowds on long spans as compared with short spans, and for country bridges as compared with those in the city, the load is varied accordingly. Each truss supports one-half of the whole load.

By multiplying the given weight per square foot by the clear width of roadway and sidewalks the live load per linear foot of bridge is obtained. This load is to be placed so as to produce the largest possible stress in any truss member considered.

The above loads for highway bridges apply to the determination of stresses in the trusses only. For the floor beams and stringers concentrated loads due to road rollers, traction engines, or heavy motor trucks should be used.

The live load for a railroad bridge is that of the heaviest cars and locomotives which pass, or are to pass over it. When a bridge is to be designed these loads are generally specified by the railroad company. In a paper entitled Rolling Loads on Bridges, by J. E. GREINER, published in Proceedings of American Railway Engineering Association, 1914, vol. 15, part 2, page 233, the live loads specified by 39 American Railroads are tabulated. Of these all except the Pennsylvania Lines west of Pittsburgh use a system of locomotive axle loadings like those described more fully in Art. 39. In 1917, the Pennsylvannia Lines are using a uniform live load per track of 5 500 pounds per linear foot, with one excess load (Art. 38) of 66 000 pounds; this load is 10 per cent heavier than that specified by the Pennsylvania Lines in 1906.

If the bridge have only one track each truss sustains but one-half the loads above given, but if it have two tracks each truss may sustain the loads as stated.

ART. 28. DEAD LOAD STRESSES.

The principle of the force polygon used in the last chapter for the determination of stresses in roof trusses may often be advantageously employed for the analysis of bridge trusses.

As an illustration let a through Pratt truss for a highway bridge be taken, having 8 panels, a span of 176 feet, and a depth of 26 feet. Let the bridge have a roadway 21 feet wide and two sidewalks each 6 feet wide. By the formula in Art. 27 the dead load per linear foot of bridge is found to be 1 627 pounds, and the dead panel load per truss is 8.95 short tons. It is required to determine the stresses due to this dead load, all being supposed to be on the lower chord.

In Fig. 54 let qy be laid off by scale equal to $8.95 \times 7 = 62.65$ tons, let it be divided carefully into 7 equal parts, lettered in

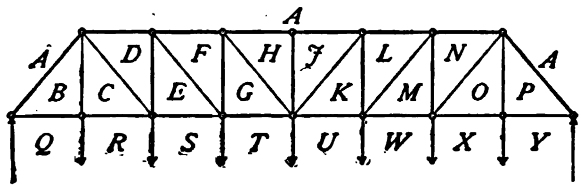


Fig. 53.

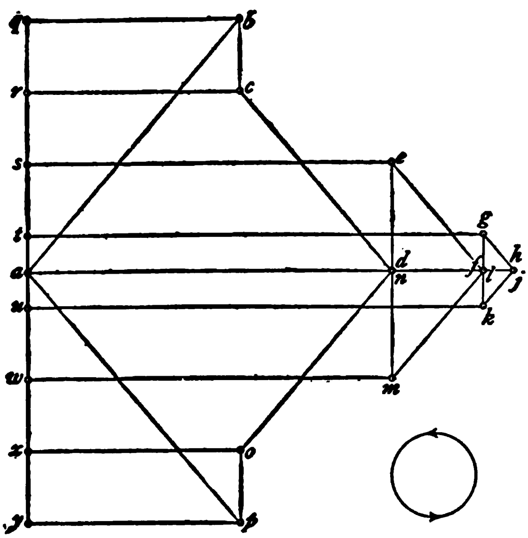


Fig. 54.

the manner indicated and bisected at a. The effective reactions are ya and aq, the half panel loads at the supports being omitted for convenience. At the left support the reaction AQ is held in equilibrium by the stresses in QB and BA, and by Art. 1 these will form the closed force triangle aqb. As aq acts in the direction from a to q the other forces must act in the same direction around the triangle, that is, from q to b and from b to a. Transferring these directions to the joint AQB the stress in QB acts away from the joint and is therefore tension, while the stress in BA acts toward the joint and is compression. The construction of the stress diagram is continued by passing to the joints alternately on the lower and upper chords until the middle of the truss is reached, then beginning at the right support and passing to the joints in the opposite direction until the diagram closes. If accurately drawn the diagram will be symmetrical with respect to *ah*. The polygon *qrcb* being a rectangle shows the tension in *CB* to be equal to the load *QR*, and the tension in *RC* to be equal to that in *BQ*. The rectangle *esad* shows the stresses in *AD* and *ES* to be equal in magnitude, the former being compression and the latter tension as previously determined. Again, *af* equals *gt* for a similar reason. It is seen therefore that the stresses in any two chord members whose adjacent spaces are separated only by a vertical have the same magnitude.

The compression in the upper chord increases toward the middle of the truss, and the same is true of the tension in the lower chord. The diagonals are all in tension but *AB*, while the verticals are all in compression but *BC*. In the web members the stresses increase from the middle toward the ends of the truss, with the exception of *BC*, which only serves to transfer the load *QR* to the upper chord.

The following results were obtained from a stress diagram drawn to a scale of 8 tons to an inch :

Truss Members.		Stresses.	Truss Members.		Stresses.
		Tons.			Tons.
Upper chord	*AD*	− 45.4	Diagonals	*AB*	− 41.1
	AF	− 56 8		*CD*	+ 29.3
	AH	− 60.6		*EF*	+ 17.6
				GH	+ 5.9
Lower chord	*BQ*	+ 26.5	Verticals	*BC*	+ 9.0
	CR	+ 26.5		*DE*	− 13.5
	ES	+ 45.4		*FG*	− 4.5
	GT	+ 56.8		*HJ*	0

As a final check the stress in *AH* is computed thus:

$$31.325 \times 88 - 8.95\,(66 + 44 + 22) + AH \times 26 = 0,$$

whence $AH = -60.6$ tons.

If instead of concentrating all the dead load on the lower chord, it be divided so that panel loads of 2.95 tons be applied on the upper chord and 6.0 tons on the lower, the stress diagram assumes the form shown in Fig. 56, the broken lines referring to stresses in the right half of the truss. The loads and reactions are taken in regular order around the truss and laid off in succession on the load line. The lower panel loads are taken from left to right, then the right reaction followed by the panel loads on the upper chord from right to left, and finally the left reaction which closes the polygon. This polygon is *efghh'g'f'e'a'b'c'd'dcbae.* The stresses obtained are marked on the right half of the truss diagram, compression being indicated by the heavy lines and tension by the light lines.

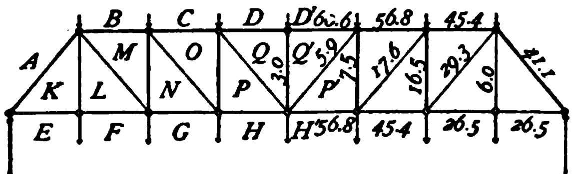


FIG. 55.

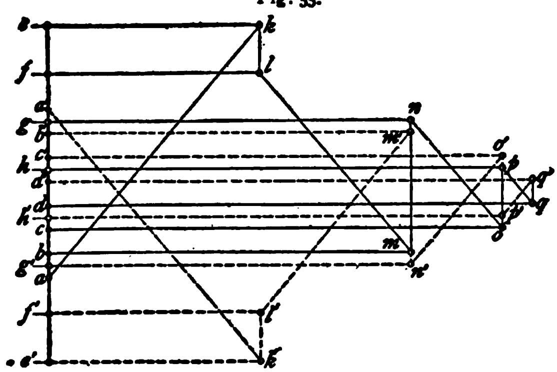


FIG. 56.

Comparing Fig. 56 with Fig. 54 it is observed that all the stresses are the same except those in the verticals whose compression is increased by 2.95 tons—the weight of the upper panel loads. The tension in *KL* is accordingly diminished by the same amount.

A further examination of Fig. 56 shows that the vertical component of *ak* is *ae*, the reaction; the vertical component of *lm* is $ae - ef - ba$; $mn = ae - ef - fg - ba$; and so on. Therefore the following principle is established:

> For trusses with horizontal chords the vertical component of the stress in any web member equals the reaction minus all the loads on the left, that is, equals the vertical shear for that member.

The only exception to this is the vertical *KL* for reasons already given in the first part of this article. The above principle may be derived from the relation existing between the stresses in any section of a truss and the external forces on either side of that section as demonstrated in Arts. 7 and 17.

The diagram also shows that the difference between the magnitudes of the stresses in any two chord members equals the sum of the horizontal components of the stresses in the web members situated between them. For instance, the difference between *hp* and *gn* is the horizontal component of *no*, which also equals the difference between *co* and *gn* or between *ph* and *bm*. The horizontal component of any diagonal is called a chord increment and forms the base of a right triangle whose height is the vertical shear in that diagonal. (Roofs and Bridges, Part I, Art. 26.)

Prob. 40. A through Pratt truss of a single track railroad bridge consists of 7 panels, each 23 feet 2 inches long and 25 feet deep. One-third of the dead load being on the upper chord, find the stresses in all the members.

ART. 29. LIVE LOAD STRESSES IN A WARREN TRUSS.

As every load placed upon a bridge truss produces compression in the upper chord and tension in the lower chord, the greatest chord stresses produced by a live load occur when every panel point of the chord supporting the floor beams is loaded. The chord stresses due to a uniform live load are hence obtained from a diagram exactly similar to that for a dead load applied only upon one chord. Hence the stress in any chord member, due to a uniform live load, bears the same ratio to the dead load stress as that of the corresponding apex loads, and accordingly either stress may be derived from the other by using this constant ratio.

In order to investigate the effect of live load on the web members, let a deck Warren truss of 7 panels be taken, the span being 126 feet, the depth 12 feet, and the live load 1 700 pounds per linear foot per truss. The live panel load is then 15.3 tons.

Placing a panel load at apex 1 in Fig. 57, the stresses due to this single load are obtained by drawing Fig. 58 in the usual

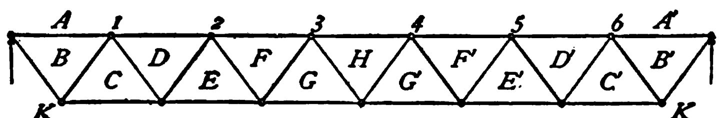


Fig. 57.

manner. The reaction $a'k$ is one-seventh of the panel load. The stresses in the braces are found to be alternately compression and tension each way from the load, and on either side the stresses are the same in magnitude from the load to the support, their vertical components being equal to the reaction on that side.

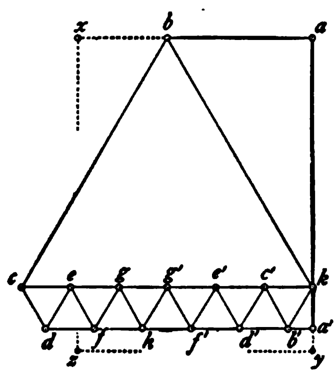


Fig. 58.

For a panel load at apex 2 the reaction of the right support will be twice as great as for the load at 1, and hence the stresses in all the braces on the right of apex 2 will also be twice as large; for a load at apex 3 the stresses on its right will be three times as great as for the load at 1, and so on. Again, a panel load at apex 6 will produce the same stresses on its left as the load at 1 caused on its right, and a load at 3 will produce stresses in the braces on its left equal to four times those due to the load at

6. The stress in each web member due to a single live panel load at any apex may therefore be obtained by taking a simple multiple of the stress for that member as given by Fig. 58.

In the following table the first and sixth lines are thus filled out directly with the results scaled off from the diagram (which was originally drawn to a scale of 5 tons to an inch), and the other lines by taking multiples of these as indicated above. The stresses in each column are then combined so as to give the greatest and least stresses and those due to a uniform live load throughout.

WEB MEMBERS.		*KB*	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*
Live panel load at	1,	+16.38	−16.38	−2.73	+2.73	−2.73	+2.73	−2.73
	2,	+13.65	−13.65	+13.65	−13.65	−5.46	+5.46	−5.46
	3,	+10.92	−10.92	+10.92	−10.92	+10.92	−10.92	−8.19
	4,	+8.19	−8.19	+8.19	−8.19	+8.19	−8.19	+8.19
	5,	+5.46	−5.46	+5.46	−5.46	+5.46	−5.46	+5.46
	6,	+2.73	−2.73	+2.73	−2.73	+2.73	−2.73	+2.73
Live load, greatest,		+57.33	−57.33	+40.95	−40.95	+27.30	−27.30	+16.35
Live load, least,		0	0	−2.73	+2.73	−8.19	+8.19	−16.35
Uniform live load,		+57.33	−57.33	+38.22	−38.22	+19.11	−19.11	0

It is found that for any given diagonal all the loads on one side of it cause one kind of stress, while those on the other side cause the opposite stress. The maximum stress is hence produced in a web member when the live load covers the larger segment of the span, and the minimum stress when the smaller segment is loaded.

In the construction of stress diagrams for a truss with horizontal chords and equal panels it is not necessary to draw the skeleton outline of the truss to a large scale. If in this example *ax* be laid off by the linear scale equal to some convenient multiple of the half panel length and *ay* equal to the same multiple of the depth of truss, *xy* will give the direction of half the web members, and in transferring this direction the triangle will require very little shifting along a straight edge, thus promoting accuracy. The line *ay* should be longer than *ak*. Completing the rectangle *xayz*, the direction of the remaining braces will be given by *az*.

The results in the line 'uniform live load' in the table should be the same as those derived from a stress diagram made for a live panel load at every panel point or apex, and may thus be checked. As such a diagram is required for the chord stresses it will also answer this purpose.

As the live load cannot act alone, but always in conjunction with the dead load, the stresses due to the combined loads are required. These are given in the following table. The dead apex load on the lower chord is 1.71 tons, and on the upper chord 3.42 tons. The dead load stresses were obtained from a diagram of 5 tons to an inch.

WEB MEMBERS.	*KB*	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*
Live load, greatest,	+ 57.33	− 57.33	+ 40.95	− 40.95	+ 27.30	− 27.30	+ 16.35
Live load, least,	0	0	− 2.73	+ 2.73	− 8.19	+ 8.19	−16.35
Dead load,	+ 20.35	− 18.18	+ 13.85	− 11.68	+ 7.44	− 5.34	+ 1.07
Maximum,	+ 77.68	− 75.51	+ 54.80	− 52.63	+ 34.74	− 32.64	+ 17.42
Minimum,	+ 20.35	− 18.18	+ 11.12	− 8.95	− 0.75	+ 2.85	− 15.28

The stresses due to the combined load are obtained by adding the dead load stresses to each of the corresponding live load stresses.

By comparing these results with computations made by the analytic method given in Part I, Arts. 21 and 22, it is seen that they are correct to or within one-tenth of a ton, which is sufficiently accurate for all purposes of design.

The same method of tabulation might be applied to the chord stresses, but the diagram for a full live load can be made in less time.

Prob. 41. Find the maximum and minimum chord stresses for the above example.

ART. 30. LIVE LOAD STRESSES IN A PRATT TRUSS.

If a Pratt truss were built having only those diagonals which are strained under dead load it would be necessary that some of them resist the compression produced by certain positions of the live load. As, however, the diagonals are only to be subjected to tension this is prevented by inserting other diagonals inclined in the opposite direction. Panels having two diagonals are said to be counter-braced and the additional diagonals are called counter-ties or counter-braces. The main and counter brace in any panel cannot both be strained at the same time by any system of loading.

When the counter-ties are called into action by the live load the stresses in the adjacent verticals are different from what they would be provided the main braces could withstand compression. This can readily be seen by changing any diagonal and making the corresponding alteration in the stress diagram.

Let the Pratt truss whose dead load stresses were determined in Art. 28 be again considered. It consists of 8 panels each 22 feet long and 26 feet deep. The total width of the bridge, including sidewalks, is 33 feet. Taking the live load at 80 pounds per square foot of floor surface the panel load per truss is

$$\tfrac{1}{2} \times 22 \times 33 \times 80 = 29\,040 \text{ pounds} = 14.52 \text{ tons.}$$

The truss diagram, Fig. 59, is drawn with the diagonals all inclined one way, the main ones being on the left of the center and the counters on the right. Placing one live panel load *RR* at apex 1 the stress diagram, Fig. 60, is constructed which gives all the stresses due to this load. Fig. 61 gives the stresses due to a live panel load at apex 7. After *rr* is laid off equal to the panel load of 14.52 tons, *ru* is marked off, by a suitable linear scale, equal to 26 feet and *rt* equal to 22 feet, then *ut* gives the inclination of the diagonals. Drawing *rs* parallel to *ut* it is found

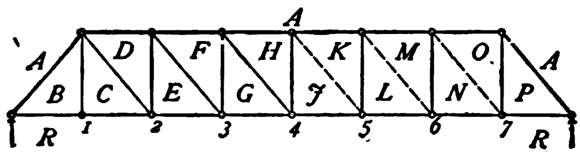


Fig. 59.

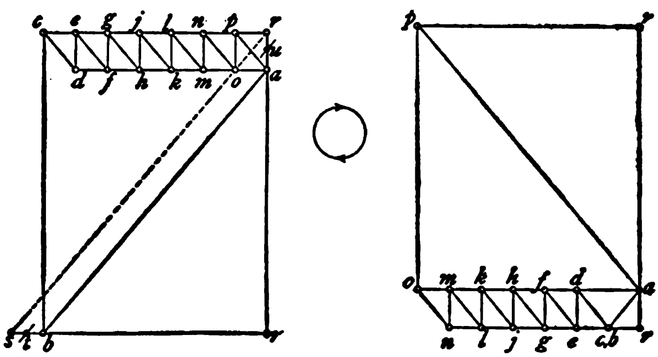


Fig. 60.

Fig. 61.

to measure 19.02 tons. In both Fig. 60 and Fig. 61 the smaller vertical lines are one-eighth of the length of *rr*, or 1.82 tons, and the smaller diagonals are one-eighth as long as *rs*, or 2.38 tons. The only part of the stress diagrams actually required consists of the similar right triangles *urt* and *rrs*, and the line *rs* is to be carefully determined. Unless these triangles are very nearly of the same size, as in the above example, the latter should be made larger than the former.

The live load stresses are then tabulated as explained in Art. 29. The dead load stresses are obtained from Fig. 62, one-half of which is like the same part of Fig. 56, while the other half is changed so as to give the stresses when the counterbraces alone are inserted in the right half of the truss as shown in Fig. 59. Two lines in this diagram are marked *hj*, the upper one measures 3.0 tons and represents the compression in *HJ* when the main ties act on each side of it (which occurs under a full live load), and the lower line measures 1.5 tons being the tension in *HJ* when the main tie acts on the left and the counter-tie on the right, as indicated in the truss diagram.

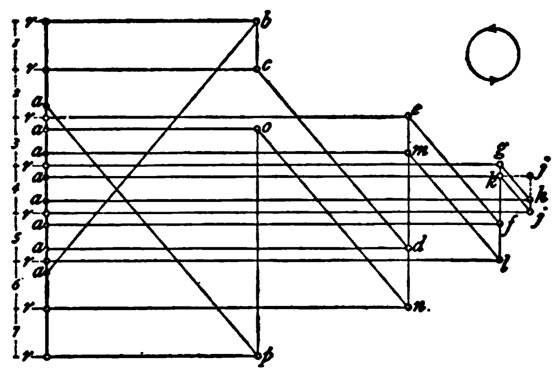


Fig. 62.

All of the tabulated results except those in the last two lines of each table were obtained as if the web members in Fig. 59

TRUSS MEMBERS.		END POSTS.	MAIN TIES.			COUNTER-TIES.		
		AB = PA	*CD*	*EF*	*GH*	*JK*	*LM*	*NO*
Live panel load at	1	− 16.7	− 2.4	− 2.4	− 2.4	− 2.4	− 2.4	− 2.4
	2	− 14.3	+ 14.3	− 4.8	− 4.8	− 4.8	− 4.8	− 4.8
	3	− 11.9	+ 11.9	+ 11.9	− 7.1	− 7.1	− 7.1	− 7.1
	4	− 9.5	+ 9.5	+ 9.5	+ 9.5	− 9.5	− 9.5	− 9.5
	5	− 7.1	+ 7.1	+ 7.1	+ 7.1	+ 7.1	− 11.9	− 11.9
	6	− 4.8	+ 4.8	+ 4.8	+ 4.8	+ 4.8	+ 4.8	− 14.3
	7	− 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4
Uniform live load,		− 66.7	+ 47.6	+ 28.5	+ 9.5	− 9.5	− 28.5	− 47.6
+ Total,		0	+ 50.0	+ 35.7	+ 23.8	+ 14.3	+ 7.2	+ 2.4
− Total,		− 66.7	− 2.4	− 7.2	− 14.3	− 23.8	− 35.7	− 50.0
Dead load,		− 41.1	+ 29.3	+ 17.6	+ 5.9	− 5.9	− 17.6	− 29.3
Maximum,		− 107.8	+ 79.3	+ 53.3	+ 29.7	+ 8.4	0	0
Minimum,		− 41.1	+ 26.9	+ 10.4	0	0	0	0

Truss Members.		Verticals.						
		BC	*DE*	*FG*	*HJ*	*KL*	*MN*	*OP*
Live panel load at	1	+ 14.5	+ 1.8	+ 1.8	+ 1.8	+ 1.8	+ 1.8	+ 1.8
	2	0	+ 3.6	+ 3.6	+ 3.6	+ 3.6	+ 3.6	+ 3.6
	3	0	− 9.1	+ 5.5	+ 5.5	+ 5.5	+ 5.5	+ 5.5
	4	0	− 7.3	− 7.3	+ 7.3	+ 7.3	+ 7.3	+ 7.3
	5	0	− 5.5	− 5.5	− 5.5	+ 9.1	+ 9.1	+ 9.1
	6	0	− 3.6	− 3.6	− 3.6	− 3.6	+ 10.9	+ 10.9
	7	0	− 1.8	− 1.8	− 1.8	− 1.8	− 1.8	+ 12.7
Uniform live load,		+ 14.5	− 21.9	− 7.3	+ 7.3	+ 21.9	+ 36.4	+ 50.9
+ Total,		+ 14.5	+ 5.4	+ 10.9	+ 18.2	+ 27.3	+ 38.2	+ 50.9
− Total,		0	− 27.3	− 18.2	− 10.9	− 5.4	− 1.8	0
Dead load,		+ 6.0	− 16.5	− 7.5	[− 3.0] + 1.5	+ 10.5	+ 19.5	+ 28.5
Maximum,		+ 20.5	− 43.8	− 25.7	− 9.4	....	....	....
Minimum,		+ 6.0	− 11.1	− 3.0	− 3.0	....	....	....

could take either tension or compression. It is now required to find the actual maximum and minimum stresses due to the combined loads under the limitation that the diagonals can take only tension. To avoid repetition only those members are referred to in the following explanation whose treatment differs from that of the preceding article.

The minimum stress in *GH* is zero as the compression due to the live panel loads at 1, 2, and 3 is greater than its dead load tension. The maximum stress in the counter *JK* is + 14.3 − 5.9 = + 8.4 tons; the minimum stress is zero since the dead load as well as the live panel loads at 1, 2, 3, and 4 tend to compress this member. A counter is therefore required in the fourth and fifth panels of the truss. The counters *LM* and *NO* are not theoretically required because the greatest tensile stress produced by the live load in each one is not sufficient to overcome the tendency of the dead load to compress the same. This is seen also from the fact that the minimum stresses in *CD* and *EF* are + 26.9 and + 10.4 tons respectively, which implies that their dead load tension is not reduced to zero under the most unfavorable position of the live load.

In finding the maximum and minimum stresses in any vertical it is necessary to consider whether the adjacent diagonals shown in the truss diagram really act under the various conditions of loading. If it is found that they do not act, then the stresses given by the table for the vertical cannot occur.

The live panel loads at 3, 4, 5, 6, and 7 together with the dead load produce in *DE* the maximum compressive stress equal to $-27.3 - 16.5 = -43.8$ tons, provided the adjacent diagonals are in tension. Under the influence of these loads *CD* and *EF* are both found to be in tension, and hence the value just obtained is the required stress. The live loads at 1 and 2 acting in addition to the dead load produce the minimum stress of $+5.4 - 16.5 = -11.1$ tons for the same reason.

The minimum stress in *FG* is due to the live panel loads at 1, 2, and 3, which with the dead load give a stress of $+10.9 - 7.5 = +3.4$ tons, provided the adjacent diagonals *EF* and *GH* are in tension. These loads would cause a tension of 22.3 tons in *EF* and 8.4 tons compression in *GH*, but as *GH* cannot take compression the counter-tie in the same panel is brought into action. The stress of $+3.4$ tons in *FG* therefore cannot occur. When the main tie acts on its left and the counter on its right, the vertical simply supports the dead panel load on the top chord, hence the minimum stress in *FG* is -3.0 tons. In the same way the minimum stress in *HJ* is also found to be -3.0 tons. Under the live panel loads at 5, 6, and 7 with the dead load the diagonals *GH* and *JK* are strained, hence the stress of -10.9 tons in *HJ* must be added to that produced by the dead load, $+1.5$ tons, to give the maximum of -9.4 tons.

Passing to the verticals on the right of the center it is seen that the combination of loads which would give either the maximum or the minimum stress according to the table will not produce tension in both of the adjacent diagonals, and accordingly no additional values can be inserted in the table. The maximum and minimum stresses for these verticals will be the same as for those on the left of the center. The tabulation for the verticals beyond *HJ* is also shown to be unnecessary as *KL* in Fig. 59 has no diagonal on its right, the counters *LM* and *NO* not being required.

The chord stresses are found in the same way as for dead load (Art. 28), only the main ties however being inserted in the truss diagram. The stresses in a Howe truss are determined in a similar manner to that employed for the Pratt truss, the diagonals in that case taking only compression.

The following important principles may now be stated, attention to which will materially reduce the work in solving other problems for either type of truss mentioned:

The maximum stress in any vertical or main diagonal is produced when the live load covers the larger segment of the span.

The maximum stress in any counter diagonal occurs when the live load covers the smaller segment of the span.

The minimum stresses in both diagonals of a counter-braced panel are zero.

The minimum stress in the diagonal of a panel not counter-braced is given when the live load covers the smaller segment of the span.

The minimum stress in any vertical adjacent to a counter diagonal equals the dead apex load on the upper chord of a Pratt truss or the lower chord of a Howe truss.

The minimum stress in a vertical not adjacent to a counter diagonal is produced when the live load covers the smaller segment of the span.

Prob. 42. A Howe truss of 12 panels for a through single track railroad bridge has a span of 123 feet and a depth of 15 feet. The dead load is 625 pounds per linear foot, one-third to be taken on the upper chord, and the uniform train load is 1 700 pounds per linear foot per truss. Find the maximum and minimum stresses.

ART. 31. SNOW LOAD STRESSES.

In addition to the dead and live load stresses must be considered those due to the snow and wind. The snow load for highway bridges is taken lower than for roofs since in the country it is not probable that the full live load would come on the bridge while a heavy fall of snow rests upon it, while in towns the sidewalks are generally cleared of snow. The snow load may vary from 20 to 0 pounds per square foot of floor surface depending upon the climate where the bridge is situated. As the floor of railroad bridges is open so that but little is retained no snow load is regarded.

As the snow load is uniform the stress diagram is exactly similar to that for dead load if the latter be taken only on the chord supporting the floor, or, like the diagram for a full live load. A separate diagram is hence not required as the stresses may be obtained from either of those mentioned by graphic multiplication, the same ratio existing between the stresses as that of the respective panel loads.

For the example in the preceding article the snow panel load is

$$\tfrac{1}{2} \times 21 \times 22 \times 15 = 3\,465 \text{ pounds} = 1.73 \text{ tons},$$

for a load of 15 pounds per square foot of floor surface for the roadway only. The ratio of snow to uniform live load stresses is therefore $\frac{1.73}{14.52} = 0.119$. This gives a snow load stress of -8.0 tons in the end post AB, $+3.4$ tons in EF, -2.6 tons in DE, etc.

Prob. 43. A through Pratt truss for a highway bridge in a village has 12 panels each 11 feet long and 14 feet deep. The roadway is 18 feet 9 inches wide, and there are two sidewalks each 5 feet wide. Find the stresses due to a snow load of 10 pounds per square foot.

ART. 32. WIND STRESSES.

The greatest stresses due to wind are produced when it blows horizontally at right angles to the line of the bridge. The surface exposed to wind action is usually taken as double the area of the side elevation of one truss. If this area be not known an approximate value may be obtained by taking as many square feet as there are linear feet in the skeleton outline of the truss. For railroad bridges the surface of the side of a train, taken at 10 square feet per linear foot of train, is added to the above. No similar addition is made for highway bridges as it is not probable that the live loads would cover them when the wind is blowing at its maximum rate.

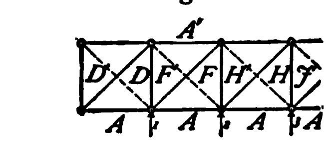


Fig. 63.

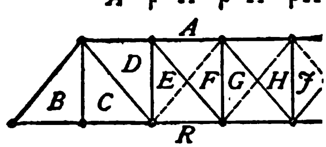


Fig. 64.

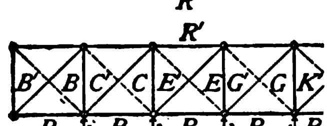


Fig. 65.

The wind pressure is taken from 30 to 40 pounds per square foot and produces its maximum effect when acting like a live load. The wind load on the trusses is divided between the upper and lower lateral bracing while the wind load upon the train is all taken by the lateral bracing of those chords which support the floor. The lateral bracing is generally of the Pratt type, the floor beams acting as the normal struts in one of the systems.

For an example let the through Pratt truss highway bridge whose dimensions are given in Art. 28 be again taken. The side elevation of the outline of the left half of the truss is shown in Fig. 64, the plan of the upper lateral bracing in Fig. 63, and that of the lower lateral bracing in Fig. 65. When the wind blows in the direction indicated by the arrows and moves from the right toward the left the diagonals drawn in full lines are strained and when it blows in the opposite direction the other set of diagonals is strained.

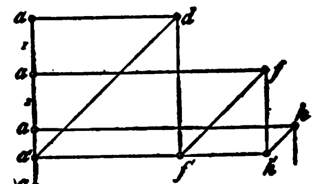


Fig. 66.

The approximate area exposed to wind action is,

$$176 + 132 + 7 \times 26 + 12 \times 34.1 = 889 \text{ square feet.}$$

Taking the wind pressure at 40 pounds per square foot the total wind load is

$$889 \times 40 = 35\,560 \text{ pounds} = 17.78 \text{ tons,}$$

and the wind panel load is

$$17.78 \div (6 + 8) = 1.27 \text{ tons.}$$

The chord stresses are determined for uniform wind load by means of Fig. 66 for the upper lateral bracing and from Fig. 67 for the lower system. Only one-half of each diagram is shown, the other half being symmetrical with it. When the wind blows in the opposite direction the chord stresses on each side of the bridge will exchange values.

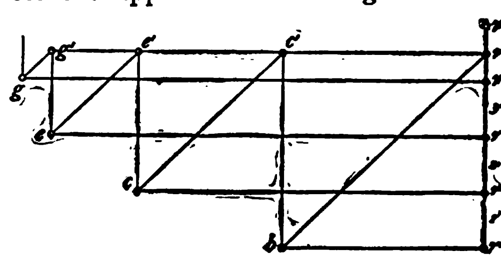


Fig. 67.

For the wind blowing in the direction of the arrows the diagrams give the following stresses in tons; for the upper chord,

AD	*AF*	*AH*	*A'F'*	*A'H'*
− 3.3	− 5.3	− 6.0	+ 3.3	+ 5.3

and for the lower chord,

RB	*RC*	*RE*	*RG*	*R'C'*	*R'E'*	*R'G'*
− 4.7	− 7.9	− 9.9	− 10.6	+ 4.7	+ 7.9	+ 9.9

In accordance with the simplified construction given in Art. 30 let a horizontal and a vertical line be drawn through *s* in Fig. 68. Let *st* be laid off equal to 21 feet and *su* equal to 22 feet, then *tus* will be the angle which the diagonals make with the chords. With a suitable scale of force let *sw* be made equal to 1.27 tons and *wx* drawn parallel to *tu*. Applying the scale to *wx* it is found to measure 1.84 tons. Dividing this value by 6 and 8, the number of panels in the upper and lower chords respectively, the quotients 0.31 and 0.23 tons are obtained which form the basis of the tabulations for the stresses in the diagonals. Dividing the panel load, 1.27 tons, by the same numbers the corresponding quotients 0.21 and 0.16 tons are found for the verticals. The following tables are now prepared:

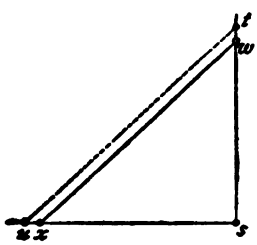


Fig. 68.

For the upper lateral bracing,

	Diagonals.			Struts.		
	DD'	*FF'*	*HH'*	*DF'*	*FH'*	*HJ'*
Wind panel load at 1	+ 1.55	....	....	− 1.05	....	....
2	+ 1.24	+ 1.24	....	− 0.84	− 0.84	....
3	+ 0.93	+ 0.93	+ 0.93	− 0.63	− 0.63	− 0.63
4	+ 0.62	+ 0.62	+ 0.62	− 0.42	− 0.42	− 0.42
5	+ 0.31	+ 0.31	+ 0.31	− 0.21	− 0.21	− 0.21
Maximum wind stresses	+ 4.7	+ 3.1	+ 1.9	− 3.2	− 2.1	− 1.3

and for the lower lateral bracing,

	Diagonals.				Struts.			
	BB'	*CC'*	*EE'*	*GG'*	*BC'*	*CE'*	*EG'*	*GK'*
Wind panel load at 1	+ 1.61	....	....	....	− 1.12	....	....	....
2	+ 1.38	+ 1.38	....	....	− 0.96	− 0.96	....	....
3	+ 1.15	+ 1.15	+ 1.15	....	− 0.80	− 0.80	− 0.80	....
4	+ 0.92	+ 0.92	+ 0.92	+ 0.92	− 0.64	− 0.64	− 0.64	− 0.64
5	+ 0.69	+ 0.69	+ 0.69	+ 0.69	− 0.48	− 0.48	− 0.48	− 0.48
6	+ 0.46	+ 0.46	+ 0.46	+ 0.46	− 0.32	− 0.32	− 0.32	− 0.32
7	+ 0.23	+ 0.23	+ 0.23	+ 0.23	− 0.16	− 0.16	− 0.16	− 0.16
Maximum wind stresses	+ 6.4	+ 4.8	+ 3.5	+ 2.3	− 4.5	− 3.4	− 2.4	− 1.6

Both diagonals in the same panel have equal stresses due to wind. The minimum stresses in all the web members are zero. Since every panel is counter-braced only tensile stresses in the diagonals and compressive stresses in the struts need to be tabulated.

Prob. 44. A through single track railroad bridge has a span of 120 feet. Its trusses are of the Pratt type, have 6 panels, 21 feet deep, and are 16 feet apart between centers. Find the stresses due to a wind pressure of 40 pounds per square foot, provided only the wind pressure on the train be considered as a moving load.

ART. 33. STRESSES DUE TO INITIAL TENSION.

In trusses whose diagonals take only tension the counterties are made adjustable in order to be drawn up to a certain degree of tension when the bridge is unloaded. The stress thus introduced in these truss members is called initial tension, and serves to prevent the vibration of the diagonals in the counter-braced panels under moving loads and to stiffen the truss as a whole.

It is required to determine the stresses produced in other members of the truss when all the counters are subjected to a given amount of initial tension. The number of counters in practice is larger than is theoretically required. As the stress in any counter is equivalent to two external forces, each equal to the initial tension, applied at the joints united by the counter-tie and acting toward each other, it may be replaced by them in this analysis.

In the Pratt truss considered in Arts. 28, 30, and 32 let the counters in the third, fourth, fifth, and sixth panels be each subject to an initial tension of 4 tons. The truss diagram is shown in Fig. 69, each of the external forces being equal to 4 tons.

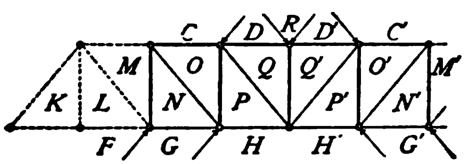


Fig. 69.

In Fig. 70 let these external forces, taken in regular order around the truss, be laid off, thus forming the closed polygon *cdrd'c'g'h'hgc*. Since each pair of forces is in equilibrium the entire system is in equilibrium and hence there are no reactions at the supports. No forces being applied at any of the joints of the first panel the members drawn in broken lines may be omitted. The stress diagram is now completed in the usual way, the characters of the stresses determined, and their magnitudes found by applying the scale, the results, expressed in tons, being as follows:

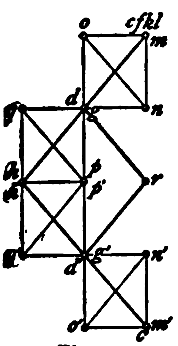


Fig. 70.

CHORDS.	STRESSES.	VERTICALS.	STRESSES.	MAIN TIES.	STRESSES.
CO and *GN*	-2.6	*MN*	-3.1	*NO*	$+4.0$
DQ and *HP*	-2.6	*OP* and *QQ'*	-6.1	*PQ*	$+4.0$

The stress diagram shows that the tension in any counter affects only the members of the panel to which it belongs. It produces compression in both chords equal to the horizontal component of the initial tension, compression in the verticals equal to its vertical component, and tension in the main tie equal to that in the counter. The general effect is therefore to increase the stresses due to other loads in all the members of the counter-braced panels except the lower chord, whose stresses are diminished. In the upper and lower lateral bracing both diagonals in each panel are made adjustable.

It is not necessary to make stress diagrams for these lateral systems, but simply to draw a right triangle whose base is parallel to the chords and whose hypothenuse, measuring 4.0 tons, is parallel to one of the diagonals. Applying the scale the base is found to measure 2.9 tons and the perpendicular 2.8 tons. The stress in the chords is therefore -2.9 tons throughout; in the end struts -2.8 tons; in the remaining struts $2(-2.8) = -5.6$ tons; and in the diagonals $+4.0$ tons. When the struts are not normal to the chords it is best to construct the complete stress diagrams.

It has not been customary, however, to consider the stresses caused by initial tension in the counters except those in the main ties. An examination of the tables in the next article will show the relation which these stresses bear to the others and to the final maximum and minimum stresses.

Prob. 45. A through double-track railroad bridge 140 feet in span has Pratt trusses of 7 panels and 32 feet deep. The bridge is 28 feet wide between centers of chords. Find the stresses due to an initial tension of 5 tons in every counter of the trusses and lateral systems.

ART. 34. FINAL MAXIMUM AND MINIMUM STRESSES.

The final maximum and minimum stresses in any truss member are the extreme limits of stress to which it is subjected by all possible combinations of the dead, live, snow, and wind loads, and initial tension. The larger limit is called the maximum and the smaller the minimum stress, and they may have the same or opposite signs. In finding the maximum and minimum stresses in the following tables, it is assumed that the initial tension as well as the dead load is always acting. In practice the wind stresses in the chords are more frequently disregarded than taken into account.

In Art. 30 the maximum and minimum stresses due only to dead and live loads were found for the through Pratt truss whose stresses due to snow load were found in Art. 31, those due to wind load in Art. 32, and to initial tension in Art. 33. The various results are now brought together in the following table and the final maximum and minimum stresses obtained by addition. The members are designated as in Figs. 63, 64, and 65.

	UPPER CHORD.			LOWER CHORD.			
	AD	*AF*	*AH*	*RB*	*RC*	*RE*	*RG*
Dead load,	− 45.4	− 56.8	− 60.6	+ 26.5	+ 26.5	+ 45.4	+ 56.8
Live load,	− 73.7	− 92.2	− 98.3	+ 43.0	+ 43.0	+ 73.7	+ 92.2
Snow load,	− 8.8	− 11.0	− 11.7	+ 5.1	+ 5.1	+ 8.8	+ 11.0
North wind,	− 3.3	− 5.3	− 6.0	− 4.7	− 7.9	− 9.9	− 10.6
South wind,	0	+ 3.3	+ 5.3	0	+ 4.7	+ 7.9	+ 9.9
Initial tension—Truss,	0	− 2.6	− 2.6	0	0	− 2.6	− 2.6
" —Lateral system,	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9
Maximum stress,	− 134.1	− 170.8	− 182.1	+ 71.7	+ 76.4	+ 130.3	+ 164.4
Minimum stress,	− 48.3	− 59.0	− 60.8	+ 18.9	+ 15.7	+ 30.0	+ 40.7

	END POST.	MAIN TIES.			COUNTER-TIE.	VERTICALS.			
	AB	*CD*	*EF*	*GH*	*GH*	*BC*	*DE*	*FG*	*HJ*
Dead and live load max.,	− 107.8	+ 79.3	+ 53.3	+ 29.7	+ 8.4	+ 20.5	− 43.8	− 25.7	− 9.4
Dead and live load min.,	− 41.1	+ 26.9	+ 10.4	0	0	+ 6.0	− 11.1	− 3.0	− 3.0
Snow load,	− 8.0	+ 5.7	+ 3.4	+ 1.1	0	+ 1.7	− 2.6	− 0.9	0
Initial tension,	0	0	+ 5.0	+ 5.0	+ 5.0	0	− 3.1	− 6.1	− 6.1
Maximum stress,	− 115.8	+ 85.0	+ 61.7	+ 35.8	+ 13.4	+ 22.2	− 49.5	− 32.7	− 15.5
Minimum stress,	− 41.1	+ 26.9	+ 15.4	+ 5.0	+ 5.0	+ 6.0	− 14.2	− 9.1	− 9.1

	Upper Lateral Bracing.					
	Diagonals.			Struts.		
	DD'	*FF'*	*HH'*	*DF'*	*FH'*	*HJ'*
Wind, Initial tension,	+ 4.7 + 4.0	+ 3.1 + 4.0	+ 1.9 + 4.0	− 3.2 − 5.6	− 2.1 − 5.6	− 1.3 − 5.6
Maximum stress, Minimum stress,	+ 8.7 + 4.0	+ 7.1 + 4.0	+ 5.9 + 4.0	− 8.8 − 5.6	− 7.8 − 5.6	− 6.9 − 5.6

	Lower Lateral Bracing.							
	Diagonals.				Struts.			
	BB'	*CC'*	*EE'*	*GG'*	*BC'*	*CE'*	*EG'*	*GK'*
Wind, Initial tension,	+ 6.4 + 4.0	+ 4.8 + 4.0	+ 3.5 + 4.0	+ 2.3 + 4.0	− 4.5 − 5.6	− 3.4 − 5.6	− 2.4 − 5.6	− 1.6 − 5.6
Maximum stress, Minimum stress,	+ 10.4 + 4.0	+ 8.8 + 4.0	+ 7.5 + 4.0	+ 6.3 + 4.0	− 10.1 − 5.6	− 9.0 − 5.6	− 8.0 − 5.6	− 7.2 − 5.6

Prob. 46. Find the maximum and minimum stresses in the chords of the above example, provided the effect of the wind be disregarded. Also, compute the greatest percentage of reduction in the maximum stress of any chord member on this account.

ART. 35. THE BOWSTRING TRUSS.

This form of truss is shown in Figs. 71, 72, and 73, and is frequently used for highway bridges. The apex points of the upper chord lie upon the arc of a circle. When the bracing is arranged like that in Fig. 71, the diagonals take only tension, while the verticals take either tension or compression. In the truss in Fig. 73, all the web members are made to sustain either kind of stress. The same is true of the form given in Fig. 72, with the exception of the middle and end verticals, which are subject to tension only.

For example, let a truss like Fig. 71 be taken whose upper panel points lie in the arc of a circle. Let it have 8 panels, each 14 feet long on the lower chord, with a depth at the center of 16 feet. The bridge has a roadway 22 feet wide and two sidewalks each 5 feet wide.

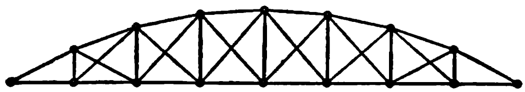


Fig. 71.

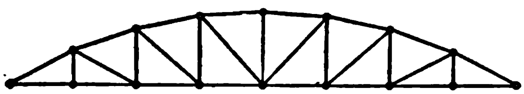


Fig. 72.

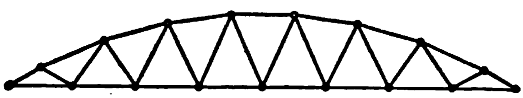


Fig. 73.

The dead panel load is found to be 4.19 tons, of which 1.40 tons is to be taken on the upper chord and 2.79 tons on the lower. The snow panel load is 1.68 tons. At 90 pounds per square foot of floor surface the live panel load is 10.08 tons, or 6 times the snow load.

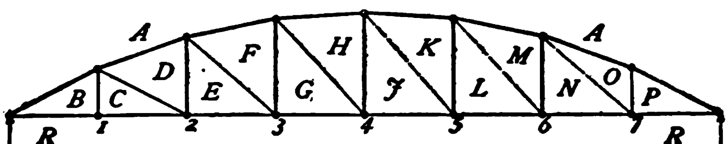


Fig. 74.

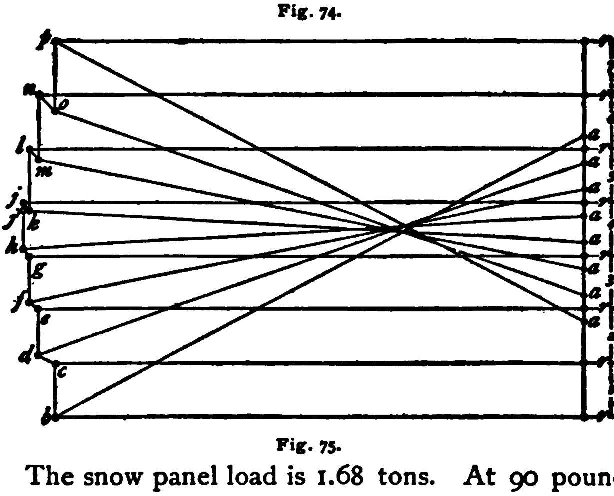


Fig. 75.

Let a truss diagram be drawn as in Fig. 74, containing only the main diagonals in the left half and the counters in the right. The depths of the truss at the first, second, and third panel points are 7.32, 12.23, and 15.07 feet respectively. The stress diagram obtained for dead load is shown in Fig. 75, that for a live panel load at apex 1 in Fig. 76, that for a live panel load at apex 7 in Fig. 77, and that for a uniform live load in Fig. 78. As a check upon the construction of these diagrams it is observed that in Figs. 75 and 78 *bc* and *op* are in the same vertical line. The same is true of *de* and *mn* and of *fg* and *kl*. In general the lines representing stresses in verticals equally distant from the center of the truss lie in the same vertical line, or are equally distant from the load line. In Fig. 76 the line *de* is at the same distance from the load line as *nm* in Fig. 77, also *nm* in Fig. 76 and *ed* in Fig. 77 are similarly situated. The same relation exists between the lines representing stresses in any two verticals occupying symmetri-

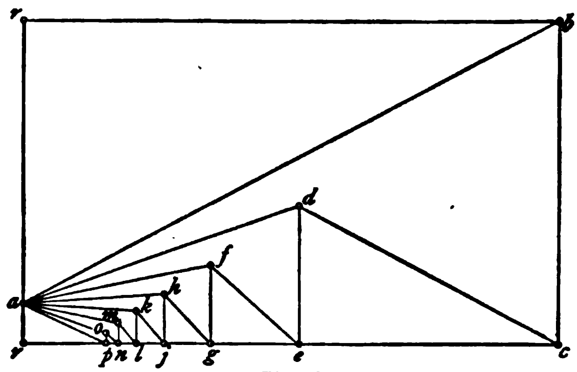


Fig. 76.

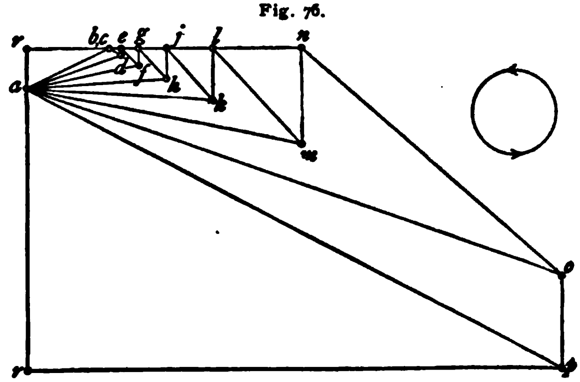


Fig. 77.

cal positions in the truss. Again, if in Fig. 77 *lk* be produced to meet *am* and the intersection be called *k'*, then *lk'* will be equal to *fg* in Fig. 76 and *nk'* will be equal to *fe* in Fig. 76.

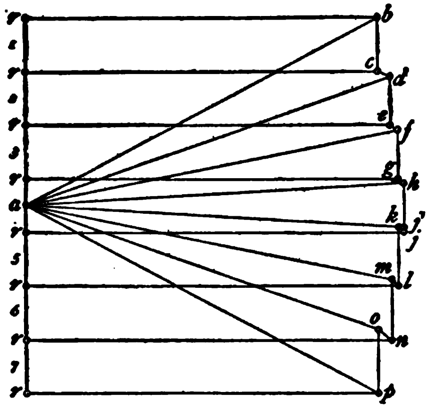


Fig. 78.

In Figs. 75 and 78 *hj'* represents the stress in *HJ* when the main diagonal is inserted on its right instead of the counter shown in Fig. 74, the point *j'* being at the intersection of the lines *ak* and *hj*. Only the stresses in web members are scaled off from Figs. 76 and 77. The snow load stresses are obtained by dividing those due to uniform live load by six.

The stress diagrams from which the following results were obtained were drawn to the following scales: The dead load diagram, 4 tons to an inch; the diagrams for single live panel loads, 2 tons to an inch; and the uniform live load diagram, 10 tons to an inch.

The results expressed in tons are now tabulated as in Art. 30 and the maximum and minimum stresses obtained, the effect of wind and initial tension being omitted.

	Upper Chord.				Lower Chord.		
	AB	*AD*	*AF*	*AH*	*RB* = *RC*	*RE*	*RG*
Dead Load,	− 31.6	− 30.5	− 29.8	− 29.4	+ 28.0	+ 28.8	+ 29.2
Live Load,	− 76.1	− 73.2	− 71.5	− 70.6	+ 67.4	+ 69.1	+ 70.1
Snow Load,	− 12.7	− 12.2	− 11.9	− 11.8	+ 11.2	+ 11.5	+ 11.7
Maximum,	− 120.4	− 115.9	− 113.2	− 111.8	+ 106.6	+ 109.4	+ 111.0
Minimum,	− 31.6	− 30.5	− 29.8	− 29.4	+ 28.0	+ 28.8	+ 29.2

		Main Diagonals.			Counters.		
		CD	*EF*	*GH*	*JK*	*LM*	*NO*
Live panel load at apex	1	− 9.3	− 3.8	− 2.1	− 1.4	− 0.9	− 0.6
	2	+ 3.2	− 7.5	− 4.2	− 2.7	− 1.8	− 1.3
	3	+ 2.7	+ 4.2	− 6.4	− 4.1	− 2.8	− 1.9
	4	+ 2.1	+ 3.3	+ 5.2	− 5.4	− 3 7	− 2.5
	5	+ 1.6	+ 2.5	+ 3.9	+ 6.6	− 4.6	− 3.2
	6	+ 1.1	+ 1.7	+ 2.6	+ 4.4	+ 8.3	− 3.8
	7	+ 0.5	+ 0.8	+ 1.3	+ 2.2	+ 4.1	+ 10.9
+ Total,		+ 11.2	+ 12.5	+ 13.0	+ 13.2	+ 12.4	+ 10.9
− Total,		− 9.3	− 11.3	− 12.7	− 13.6	− 13.8	− 13.3
Uniform live load,		+ 1.9	+ 1.2	+ 0.3	− 0 4	− 1.4	− 2.4
Dead load,		+ 0.8	+ 0.6	+ 0.2	− 0.2	− 0.6	− 1.0
Snow load,		+ 0.3	+ 0.2	+ 0.1	− 0.1	− 0.2	− 0.4
Maximum,		+ 12.3	+ 13.3	+ 13.3	+ 13.0	+ 11.8	+ 9.9
Minimum,		0	0	0	0	0	0

		Verticals.						
		BC	*DE*	*FG*	*HJ*	*KL*	*MN*	*OP*
Live panel load at apex	1	+ 10.1	+ 4.3	+ 2.5	+ 1.6	+ 1.0	+ 0.7	+ 0.4
	2	0	+ 8.6	+ 4.9	+ 3.1	+ 2.0	+ 1.3	+ 0.8
	3	0	− 1.2	+ 7.3	+ 4.6	+ 3.1	+ 2.0	+ 1.2
	4	0	− 1.0	− 2.2	+ 6.2	+ 4.1	+ 2.7	+ 1.6
	5	0	− 0.7	− 1.6	− 2.9	+ 5.1	+ 3.4	+ 2.1
	6	0	− 0.5	− 1.1	− 1.9	− 3.3	+ 4.0	+ 2.5
	7	0	− 0.2	− 0.5	− 1.0	− 1.6	− 3.0	+ 2.9
+ Total,		+ 10.1	+ 12.9	+ 14.7	+ 15.5	+ 15 3	+ 14.1	+ 11.5
− Total,		0	− 3.6	− 5.4	− 5.8	− 4.9	− 3.0	0
Uniform live load,		+ 10.1	+ 9.3	+ 9.3	[+ 9.3] + 9.7	+ 10.4	+ 11.1	+ 11.5
Dead load,		+ 2.8	+ 2.4	+ 2.4	[+ 2.5] + 2.6	+ 3.0	+ 3.3	+ 3.4
Snow load,		+ 1.7	+ 1.5	+ 1.5	[+ 1.5] + 1.6	+ 1.7	+ 1.9	+ 1.9
Maximum,		+ 14.6	+ 13.4	+ 13.2	+ 13.3	+ 12.1	+ 12.3	+ 14.4
Minimum,		+ 2.8	− 1.2	− 3.0	− 3.2	− 1.9	+ 0.3	+ 3 4

The student will have no difficulty in finding the maximum and minimum stresses in the chords and diagonals. It will be noticed that counters are required in every panel of this truss.

The application of the principles employed in Art. 30, for determining the maximum and minimum stresses in the verticals of trusses with counterbraced panels may be further illustrated by finding those in DE. The greatest compression in this vertical is shown by the table to be due to the live panel loads 3 to 7 inclusive, combined with the dead load, giving a stress of $-3.6 + 2.4 = -1.2$ tons. This stress is a real one because the adjacent diagonals CD and EF shown in Fig. 74 are then acting, both of them receiving almost their maximum stress.

The tension of 12.9 tons given in the line marked '+ total' when combined with that due to the dead and snow loads cannot actually occur because the adjacent diagonals are not brought into action by the corresponding loads. The greatest tension in DE will therefore occur under the full live, dead, and snow loads, unless one or more of the live panel loads on the right may be removed without causing either CD or EF to cease acting. If the live load be continuous, as it is customary to regard it, the live panel load 7 may be removed under the above conditions, and the corresponding stress will be $+9.3 + 2.4 + 1.5 - (-0.2) = +13.4$ tons. If the live panel loads 6 and 7 be removed, the diagonal EF ceases to act since its stress would become $+1.2 + 0.6 + 0.2 - (+1.7 + 0.8) = -0.5$ tons. Were it allowable to consider the live load as discontinuous, CD and EF would still be in tension after removing the live panel load 6 only, thus giving a stress in DE of $+9.3 + 2.4 + 1.5 - (-0.5) = +13.7$ tons.

In a similar manner the values of $+12.3$ and $+0.3$ tons are obtained for MN, and these are its maximum and minimum stresses under the condition that the adjacent counter ties LM and NO are both acting. On account of the symmetry of the truss the maximum and minimum stresses in DE have the same values provided the counter ties are acting in each adjacent panel.

Two more conditions for DE require attention. The first is that when the main diagonal acts on its right and the counter on its left. The table indicates that this condition cannot exist under any combination of the given loads. The second condition occurs when the main tie acts on the left of *DE* and the counter on its right. This one is possible, and requires an additional tabulation.

The live panel load at apex 1 produces a tension in *DE* of 1.28 tons, and that at apex 7 of 0.43 ton. The former value is obtained from Fig. 76 by measuring the distance from *d* to the point where the vertical *de* meets *af* produced, while the latter is obtained from Fig. 77 by measuring the distance from *d* to the intersection of *af* with the vertical *de* produced In the same way, Fig. 75 gives the corresponding dead load stress of + 2.8, and Fig. 78 (after dividing by six) the snow load stress of + 1.7 tons. From the stresses due to the live panel loads at 1 and 7 those produced by the loads at apexes 2 to 6 inclusive are found by the method used in the above tabulation to be + 2.6, + 2.1, + 1.7, + 1.3, and + 0.9, when expressed to the nearest tenth of a ton. Since all of these stresses are tension, it is clear that the maximum will be caused by the dead and snow loads combined with as many of the live panel loads as possible without bringing the main tie on the right of *DE* into action. The table for the diagonals indicates that this occurs when the live panel loads are placed at apexes 1 to 5 inclusive, and the resulting stress in *DE* is $+ 1.3 + 2.6 + 2.1 + 1.7 + 1.3 + 2.8 + 1.7 = +$ 13.5 tons. If a similar tabulation were made for *MN* when the counter tie acts on its left and the main tie on its right, the same result of + 13.5 tons would be obtained.

On comparing the maximum stresses in *DE* under the three conditions above described and investigated, the last value obtained is seen to be the greatest in magnitude, and hence the true maximum to be used for both *DE* and the corresponding vertical *MN* in the other half of the truss. As the range of stress from this maximum of + 13.5 to − 1.2, the minimum in *DE*, is greater than to the minimum of + 0.3 in

MN, the true minimum stress to be used for both *DE* and *MN* is — 1.2 tons.

The stresses in *FG* when the main tie acts on the left and the counter on the right are found to be + 0.8, +1.6, + 2.4, + 1.9, + 1.4, + 1.0, and + 0.5 for the live panel loads 1 to 7 inclusive, + 2.6 for the dead load, and + 1.6 tons for the snow load. The maximum occurs when the live load is at the apexes 1 to 6 inclusive, combined with the dead and snow loads, its value being + 13.3 tons. On comparing this stress with the values given in the table for *FG* and *KL*, the true maximum for both of these verticals is seen to be + 13.3 tons, and the greatest range of stress to the values of their minimum stresses shows that the true minimum for both verticals is — 3.0 tons.

A similar tabulation might be made for *HJ*, but this is unnecessary since the main ties act on each side of it under the full load. The required stresses were obtained directly from Figs. 75 and 78 as previously explained, and are inserted in the table in brackets. The maximum and minimum stresses are caused by the full load. The largest tension which can occur in *HJ* when the main diagonal acts on the left and the counter on the right is 12.3 tons. The true maximum and minimum stresses of *OP* are equal to those of the vertical *BC*.

The form of truss shown in Fig. 79 is known as the lenticular truss and is also used for highway bridges. The panel points of the chords may lie on arcs of circles, but generally are placed on parabolas. The broken lines show the roadway and its connection to the trusses, the vertical end pieces being heavy posts and the others tension rods. In determining the stresses the same method is pursued as for the bowstring truss.

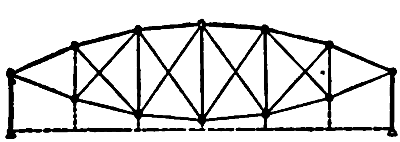


Fig. 79.

Prob. 47. A through bowstring truss has six panels each 15 feet long, the depth at the first and fifth panel points being 7.5 feet, at the second and fourth panel points 11.7 feet, and at the center 13 feet. The dead panel load is 2.5 tons and the live panel load 7.5 tons. Find the maximum and minimum stresses due to these loads only.

ART. 36. THE PARABOLIC BOWSTRING TRUSS.

When the panel points of the broken chord of a bowstring truss lie upon a parabola whose vertex is midway between the supports, the stress diagrams become simpler. Let a parabolic bowstring truss be taken with the same general dimensions and loads as given in the preceding article. In the diagrams like Figs. 75 and 78 the broken lines *bcd* . . . *nop* become straight lines, and the points *c* and *d*, *e* and *f*, . . . *n* and *o*, coincide. This shows that under a uniform load the stress in the horizontal chord is the same throughout, the diagonals are not strained at all, and each vertical carries only the panel load on the horizontal chord. In the diagrams similar to Figs. 76 and 77, the points *b*, *d*, *f*, *h*, *k*, *m*, and *o* lie upon a straight line which intersects the load line produced at the shorter distance *ar* from *r*, thus checking the construction.

In the tabulated stresses for the webbing the sum of the '+ total' and the '− total' will give zero for the diagonals and + 10.08 for the verticals provided the work be done with the utmost precision. With diagrams like Figs. 76 and 77 made to a scale of 3 tons to an inch, the stresses obtained by tabulation for uniform live load averaged 0.05 tons in magnitude for the diagonals, three being tension and three compression, and those in the verticals varied on an average 0.01 tons from the true result, some being too large and others too small.

The final results in tons are given in the following table:

CHORDS.	MAXIMUM STRESSES.	MINIMUM STRESSES.	DIAGONALS.	MAXIMUM STRESSES.	VERTICALS.	MINIMUM STRESSES.
AB	− 124.9	− 32.8	*CD*	+ 9.9	*BC*	+ 2.8
AD	− 118.6	− 31.2	*EF*	+ 11.6	*DE*	− 0.4
AF	− 114.1	− 30.0	*GH*	+ 12.9	*FG*	− 2.3
AH	− 111.9	− 29.4	*JK*	+ 13.4	*HJ*	− 2.9
					KL	− 2.3
RB, *RC*, *RE*, *RG*	+ 111.7	+ 29.3	*LM*	+ 12.9	*MN*	− 0.4
			NO	+ 11.6	*OP*	− 2.8

The minimum stresses in the diagonals are zero, and the maximum stress in each vertical equals

$$2.79 + 10.08 + 1.68 = 14.55 \text{ tons},$$

or the sum of the dead, live, and snow panel loads.

If this truss were used as a deck bridge the maximum stresses in the verticals would be those given in the accompanying table, while the minimum stresses would equal the dead panel load on the horizontal chord, or − 2.79 tons. It will be observed that the differences between the minimum stresses in the verticals of the through truss and the maximum stresses in the same members of the deck truss equal twice the dead panel load plus the live and snow panel loads. The stresses in the remaining members are the same for a deck as for a through bridge except that the chord stresses change in character.

VERTICALS.	MAXIMUM STRESSES.
BC and *OP*	− 14.5
DE and *MN*	− 17.7
FG and *KL*	− 19.6
HJ	− 20.2

The properties of the parabola are such as to provide a very simple and abridged construction for obtaining directly the maximum and minimum stresses due to dead, live, and snow loads.

Let S be the stress in the horizontal chord due to the total uniform load W (including the half panel loads at the supports), l the span and d the depth of the truss at the center, then (Roofs and Bridges, Part I, Art. 39),

$$S = \frac{Wl}{8d}.$$

Substituting for these terms their values for the truss considered above, the stress due to live load is

$$S = \frac{8 \times 10.08 \times 112}{8 \times 16} = 70.56 \text{ tons.}$$

Similarly the stress due to dead load is 29.33 tons and that due to snow load is 11.76 tons.

Now in Fig. 80 on the horizontal line ad with a scale of 4 or 5 tons to an inch, let ab be laid off equal to 29.33 tons, bc equal to

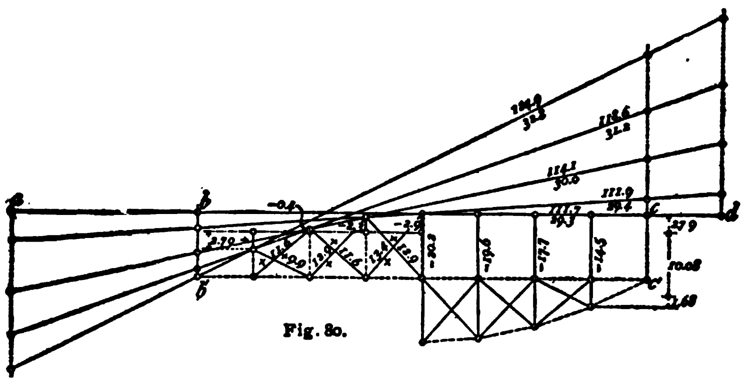


Fig. 80.

70.56 tons, cd equal to 11.76 tons, and verticals erected at each point of division. As the depth of the truss is one-seventh of its span let bb' and cc' be made equal to one-seventh of 70.56 or 10.08 tons, and on the span $b'c'$ let an outline diagram be drawn similar to the truss diagram. In the figure one-half is drawn as a through and the other half as a deck truss. By measuring the diagonals with the scale of force their maximum stresses are obtained. Their minimum stresses are zero.

Let the chord members be prolonged until they meet the verticals through *a* and *d*. Each of these lines is divided into three parts by the four verticals, these parts giving the stresses due to dead, live, and snow load respectively. For example, the dead load stress in the horizontal chord is represented by the part *ab*, the live load stress by *bc*, and the snow load stress by *cd*. The maximum stress in the same member is hence *ad*, or 111.7 tons, and the minimum stress is *ab*, or 29.3 tons.

On the through truss draw the horizontal broken and dotted base line 2.79 tons (the value of the dead panel load) above the first panel point on the upper chord. By measuring the verticals extending from this base line to each panel point on the upper chord, upward being compression and downward tension, the minimum stresses in the verticals are found. Their maximum stresses are each equal to the sum of the dead, live, and snow panel loads, or 14.55 tons.

On the deck truss let a similar base line be drawn 14.55 tons above the first panel point from the support on the lower chord, and the verticals measured from the panel points to the base line; thus are found the maximum stresses in the verticals, all of them being compression. Their minimum stresses are each equal to − 2.79 tons. The measured stresses are marked on the different lines of the diagram.

Prob. 48. A deck parabolic bowstring truss of 10 panels has a span of 120 feet and a depth of 15 feet at the center. Find the maximum and minimum stresses for a dead panel load of 3 tons and a live panel load of 7 tons.

ART. 37. APPLICATION OF THE EQUILIBRIUM POLYGON.

In the preceding articles of this chapter the method of the force polygon has been employed exclusively. To illustrate the application of the equilibrium polygon in the determination of stresses let the same example used in Art. 28 be taken, it being required to find the chord stresses, and afterward the web stresses due to the dead load. The span is 176 feet, the depth 26 feet, and the dead panel load 8.95 tons.

Let the truss diagram be drawn to a scale of 10 feet to an inch and the panel loads laid off successively on the load line *qy* in Fig. 81 to a scale of 10 tons to an inch (considerably re-

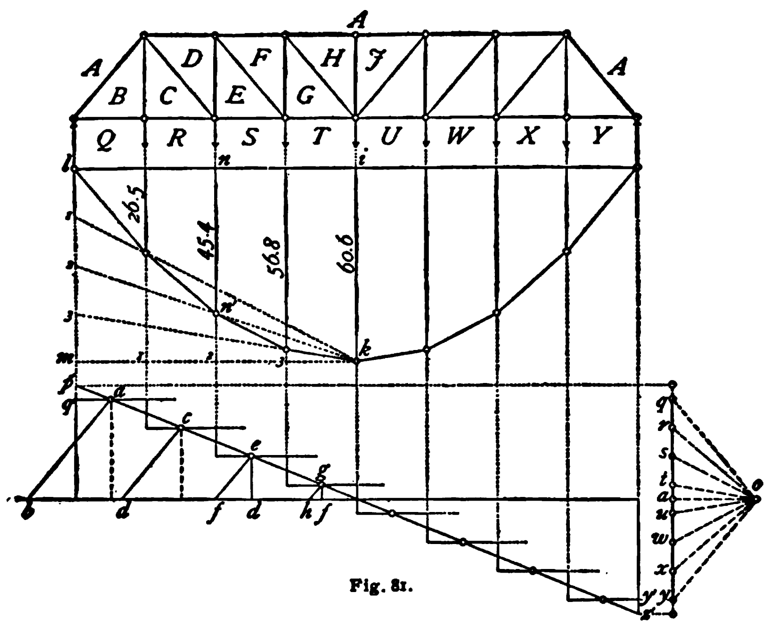


Fig. 81.

duced as here printed). The effective reactions are *ya* and *aq*, the load line being bisected at *a*. Let the pole *o* be taken on a horizontal through *a*, the pole distance *H* be made equal to 26 tons, and the equilibrium polygon constructed (Art. 6). The ordinates at the vertices of this polygon when measured by the linear scale and multiplied by *H* give the bending moments in tons-feet at the corresponding sections of the truss (Art. 7). The chord stresses are obtained by dividing these moments by 26 feet, the depth of the truss. For instance, the ordinate nn' measures 45.4 feet, whence the stress in AD or ES is

$$\frac{45.4 \times 26}{26} = 45.4 \text{ tons.}$$

The stresses may therefore be directly obtained by measuring the ordinates with a scale of 10 tons to an inch, the results being marked on the diagram. Even with a smaller scale than that indicated above the same stresses, measured to tenths of a ton, are obtained as in Art. 28.

If the linear scale be 20 feet to an inch, and H be taken as 52 tons, the scale of tons remaining the same, the ordinates should be measured by a scale of $20 \times \frac{52}{26} = 40$ tons to an inch to obtain the chord stresses. Again, if H be laid off by the linear scale equal to double the depth of the truss, then the ordinates are to be measured by double the scale of force or 20 tons to an inch.

As the vertices of the equilibrium polygon lie upon a parabola whose vertex is at k, the ordinates may be obtained without drawing the equilibrium polygon. The chord stress at the center of a truss uniformly loaded is

$$S = \frac{Wl}{8d},$$

in which W includes the half panel load at each support. Let the middle ordinate ik be made equal to

$$S = \frac{8 \times 8.95 \times 176}{8 \times 26} = 60.6 \text{ tons,}$$

let lm be made equal to ik and divided into the same number of parts as mk, in this case four. Drawing radial lines from k to these points of division their intersections with the corresponding verticals give the required points.

For an odd number of panels in a truss lm should be divided into as many parts as there are panels in the entire truss, only the alternate points of division from *l* to *m* being used. For a uniform live load the same method may be employed as that here given, or the dead and live loads may be combined in one diagram.

The shear diagram for dead load is shown below the moment diagram in Fig. 81, the ordinates representing the vertical shear being limited by the line forming a series of steps from q' to y'. If the load were not concentrated at the panel points but uniform throughout the ordinates for shear would be measured to the straight line $p'z'$, which intersects the former at the center of each panel. The inclined line is the most convenient to use, but usually it is not employed, as the analytic method, consisting only of a few subtractions or additions, is more rapidly applied. The lines *ab*, *cd*, *ef*, and *gh* are the stresses in the diagonals, and *de* and *fg* in two of the verticals. To avoid confusing the diagram *cd*, *ef* and *fg* are turned toward the left, but have the same inclination as the diagonals of the truss. The stress in *BC* is one panel load, and that in *HJ* is zero. If 2.95 tons of the dead panel load be taken on the upper chord, a compression of that amount is to be added to each of the above stresses in the verticals.

In order to determine the maximum live load shear in any panel another shear diagram is necessary. On the horizontal axis *AG* in Fig. 82 let the positions of the panel loads be marked, and their magnitudes laid off on the load line *ag*. Let the pole *o* be placed in a horizontal line through the beginning of the load line, the pole distance made equal to the span of the truss by the linear scale and the equilibrium polygon $A'B'C'$ .. H' constructed. Now let the span be so placed that its right support shall come at *F*, then every panel point from 3 to 7 inclusive is loaded. By Art. 29 this position of the load gives the maximum live load shear in *EF* or *DE* of Fig. 59. The ordinate $F'F''$ is equal to the reaction of the left support and hence equals the vertical shear in the members named; for, the ordinate being contained between the sixth side of the equilibrium polygon and the first side produced measures the sum of the moments of all the loads between them with reference to the section through F' (Art. 7). Calling the value

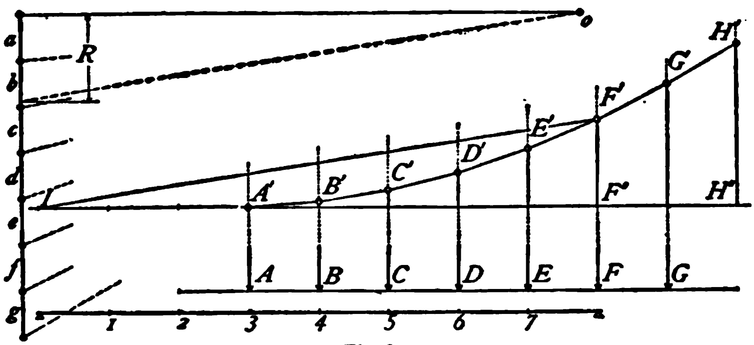


Fig. 82.

of the ordinate y, the sum of these moments equals $y \times H$. But the section at F' is at the right support of the truss, and hence the sum of the moments also equals the moment of the left reaction with reference to this support. Therefore

$$y \times H = R \times l,$$

and since H was made equal to l,

$$y = R.$$

This may also be proved by drawing through the pole a parallel to the closing line $F'I$ of the equilibrium polygon forming a triangle which is equal to $IF'F''$ since one side is equal to its parallel IF'' and all the sides of both triangles are mutually parallel. $F'F''$ is hence equal to its parallel R.

The ordinates taken in succession from H' to A' measured by the scale of force give the maximum live load shears in each panel of the truss from left to right. The stresses in the diagonals are obtained from these shears in the manner indicated in Fig. 81. The results by this method are found to be the same as those given in Art. 30 in the line '+ total' for the diagonals and in the line '— total' for the verticals.

For trusses with inclined chords the moment diagram gives only the horizontal component of any chord stress, the ordinate being measured in a section passing through the center of moments of the chord member. The shear diagram is not applicable in such cases except for the purpose of finding the reaction from which the stress in the required web member may be found by the method of the force polygon.

It will be observed by the student that the method of the equilibrium polygon does not indicate the character of the stresses as in the method of the force polygon. Whether a web member be in tension or compression is to be determined by cutting it by a plane, noting its direction and the sign of the shear, as was done in the analytic method in Part I, Art. 26.

Prob. 49. Find the maximum and minimum stresses due to dead and live loads for the truss in Prob. 42.

ART. 38. EXCESS LOADS.

It may be specified that a truss shall be designed to carry a given load extending over a certain distance in excess of the uniform live load. In a railroad bridge the excess load would represent the difference between the locomotive panel load and the uniform train panel load.

The chord stress due to a single load P distant x from the left support is a maximum at the load, and for any position of the load

$$S = \frac{M}{d} = \frac{P(l-x)x}{ld};$$

M being the bending moment at the load. If S be an ordinate corresponding to an abscissa x this is the equation of a parabola whose vertex is at the center of the truss. The middle ordinate has a value of $\frac{Pl}{4d}$ and the ordinates are zero at each end. The ordinate at each panel point is the maximum stress in the chord member whose center of moments is at that point.

Although excess loads are not used in determining the stresses in highway bridge trusses, except for the floor system, yet as an illustration of the method let the Pratt truss in Art. 30 be taken and the maximum chord stresses be found for one excess panel load of 5 tons. The ordinates in Fig. 84 are obtained as in the preceding article, the middle ordinate fk being

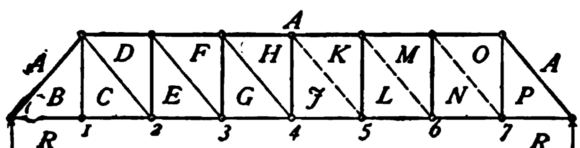


Fig. 83.

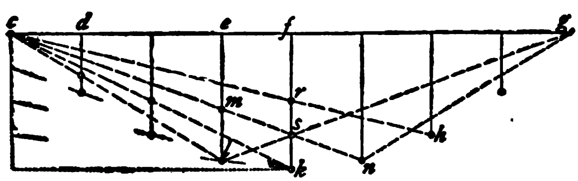


Fig. 84.

$$S = \frac{Pl}{4d} = \frac{5 \times 176}{4 \times 26} = 8.46 \text{ tons.}$$

The other ordinates on either side are 3.7, 6.4, and 7.9 tons.

Let it now be supposed that two excess loads of 5 tons each and 50 feet apart are specified. Let their distance apart be taken as 44 feet or two panel lengths. For two equal loads the maximum stress in a chord member in the left half of the truss occurs when one load is at the section passing through its center of moments and the other load is toward the right. For the member AF one load is to be placed at apex 3 and the other at 5. The stress in AF due to the load at 3 is el which measures 7.9 tons. The stresses for the load at 5 are given by the ordinates of the triangle cng, and since cn intersects el at m, the stress in AF due to this load is em, or 4.8 tons, and that due to both loads is $7.9 + 4.8 = 12.7$ tons. In a similar manner the other chord stresses are found. The stress in AH may however be larger when the loads are at 3 and 5, than when at 4 and 6, being in this example $6.4 + 6.4 = 12.8$ tons for the former and $8.5 + 4.2 = 12.7$ tons for the latter position, fs being 6.4 tons and fr 4.2 tons.

By inspecting the table in Art. 30 it is seen that the maximum shear in any web member in the left half of the truss due to a single live panel load occurs when the load is placed at the nearest panel point on its right, the shear being equal to the left reaction. For a single excess load P the vertical shear is

$$V = R = \frac{P(l - x)}{l};$$

if V be an ordinate corresponding to an abscissa x this is the equation of a straight line. Thus, when $x = 0$, $V = P$ and when $x = l$, $V = 0$. From these data the diagram for maximum shear due to a rolling load may be readily drawn.

For the above example let the span ab be laid off in Fig. 85, marking the panel points as indicated and erecting verticals at

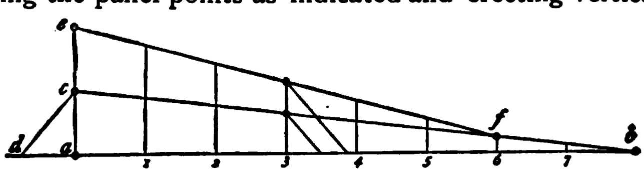


Fig. 85.

all of these points. Let ca be made equal to 5 tons by scale and cb joined, then the ordinates to this line give the maximum shear for each panel. The ordinate at 3 which measures 3.1 tons is the maximum shear in EF and DE and also the stress in DE. The stress in EF is given by a line drawn through the upper extremity of this ordinate with the same slope as EF and is found to be 4.1 tons. The stresses in all the diagonals may be obtained from $cd = 6.56$ tons by the use of a simple ratio which for EF is $\frac{5}{8}$ and for HG is $\frac{1}{8}$. These stresses may also be obtained directly from the tabulation in Art. 30, by multiplying the greatest live load stress in any member due to a single panel load by the ratio of the excess load to the live panel load. For example, the stress in *EF* is $+ 11.9 \times \frac{5.0}{14.52} = + 4.1$ tons and that in *DE* is $- 9.1 \times \frac{5.0}{14.52} = - 3.1$ tons. The same method might be applied to the chords, but would require more work than that used in this article.

For the two excess loads mentioned above the maximum stress in *EF* or *DE* is produced by placing one load at 3 and the other at 5. Let *ce* be drawn equal to $\frac{5}{8} \times 5$ tons and *e* joined with *f* which is on the line *bc* at a distance of two panels from *b*. The ordinates to the line *efb* give the maximum shears in each panel due to the two excess loads. The stress in *DE* is found to be 5.0 tons and in *EF* 6.6 tons.

The chord stresses are as follows,

	RB and *RC*	*AD* and *RE*	*AF* and *RG*	*AH*
First excess load,	3.7	6.4	7.9	8.5
Addition for second load,	2.6	4.2	4.8	—
Two excess loads,	6.3	10.6	12.7	2 × 6.4 = 12.8

The stresses in the verticals are,

	BC	*DE*	*FG*	*HJ*	*KL*	*MN*	*OP*
First excess load,	+ 5.0	− 3.1	− 2.5	− 1.9	− 1.3	− 0.6	0
Addition for second load,	0	− 1.9	− 1.3	− 0.6	0	0	0
Two excess loads,	+ 5.0	− 5.0	− 3.8	− 2.5	− 1.3	− 0.6	0

and those in the diagonals are,

	AB	*CD*	*EF*	*GH*	*JK*	*LM*	*NO*
First excess load,	− 5.7	+ 4.9	+ 4.1	+ 3.3	+ 2.5	+ 1.6	+ 0.8
Addition for second load,	− 4.1	+ 3.3	+ 2.5	+ 1.6	+ 0.8	0	0
Two excess loads,	− 9.8	+ 8.2	+ 6.6	+ 4.9	+ 3.3	+ 1.6	+ 0.8

These results when combined with the stress given in Art. 34 will increase the maximum stresses in all the members and reduce some of the minimum stresses in the web members. The addition of the stresses in the web members due to excess loads should properly be made in the tables in Art. 30 by inserting a line above the dead load stresses and then obtaining the maximum and minimum stresses anew.

If one or more locomotive panel loads be required to precede the train on a railroad bridge, the maximum chord stresses in the left half of the truss will be obtained by placing the excess loads as near as possible to the left support.

Prob. 50. A Pratt truss for a deck single track railroad bridge has 6 panels each 13 feet 4 inches long and 13 feet 4 inches deep. Find the stresses due to one excess load of 6.5 tons per truss.

CHAPTER IV.

LOCOMOTIVE AXLE LOADS.

ART. 39. STANDARD TYPICAL LOADS.

A uniform live load which is carried by the stringers and floor beams to the panel points of the trusses, giving uniform live panel loads throughout, is confined mainly to highway bridges. For railroad bridges it is generally specified that the live load shall consist of two typical consolidation locomotives, followed by a uniform train load of a given weight per linear foot of track. The given axle loads constitute a system of concentrated loads whose relations remain unchanged to each other and to the uniform load following them while passing over the bridge.

Fig. 85*a* represents two typical locomotives followed by a uniform train load according to COOPER'S standard loading,

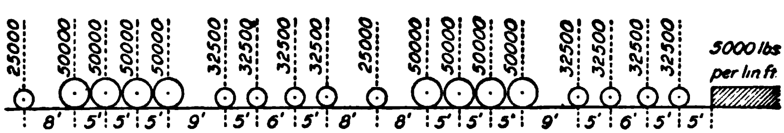


Fig. 85*a*.

Class E 50. The numbers above the wheels show the axle loads in pounds and the numbers between them show their distances apart in feet. Other classes of COOPER'S series have the same spacing of axles, while the loads on the corresponding axles are directly proportional to their class numbers. Alternate loads on two axles, 7 feet apart, are also specified for each class, the load on each of the two axles being 62 500 pounds for E 50 loading. See also Art. 40 in Roofs and Bridges, Part I.

In his treatise entitled "Bridge Engineering," J. A. L. WADDELL proposes a new series of locomotive loads which differs from COOPER'S standard only by making the axle loads on the tenders equal to 70 per cent of those of the drivers, and by increasing the spacing between the last driver axle and the first tender axle to 10 feet, and by having the uniform load preceding as well as following the concentrated loads. The alternate axle loads are 20 per cent larger than the corresponding driver axle loads.

In a paper entitled "Rolling Loads on Bridges," by J. E. GREINER, published in Proceedings of the American Railway Engineering Association, 1914, Vol. 15, Part 2, page 233, there are tabulated the live loads used by 39 railroads in the United States. Of these 39 roads all except the Pennsylvania Lines west of Pittsburgh employ a system of concentrated locomotive loads. See also the references on live loads for railroad bridges in Chap. IX.

The following systems of loadings were in the third edition of this volume and are retained because they are used in some

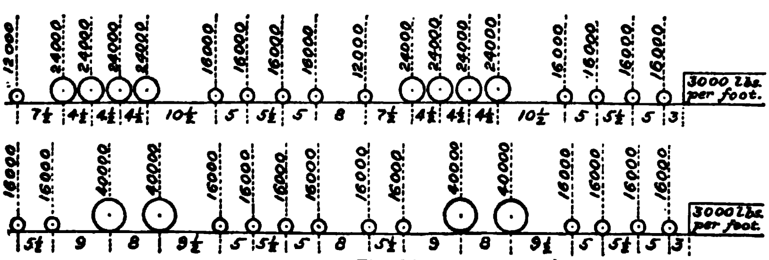


Fig. 86.

of the examples and problems. Fig. 86 represents two consolidation locomotives and passenger locomotives, followed respec ively by a uniform train load as specified in 1886 by the Pennsylvania Railroad. Fig. 87 shows two consolidation locomotives specified by the Lehigh Valley Railroad in 1889. Fig. 88 gives the locomotive and train loads for Class T of

WADDELL'S compromise standard system, which was first published in 1893. The other classes are designated as U, V,

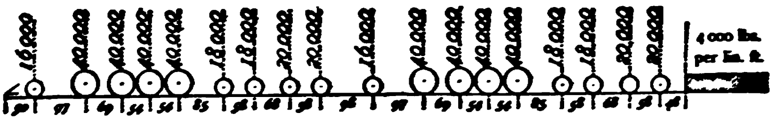


Fig. 87.

W, X, Y, and Z, all having the same spacing. The axle loads for each class are derived from those of the preceding class by

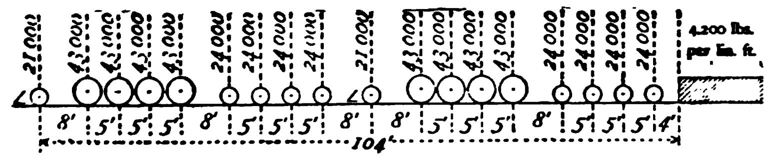


Fig. 88.

subtracting the constant difference of 1 000 pounds for the pilot axle and each tender axle, 3 000 pounds for each driver axle, and 200 pounds for the uniform train load per linear foot.

A typical locomotive does not really exist, but is used so that the stresses due to it will be greater than those due to any locomotives that are likely to be built for some years in the future. See Manual of American Railway Engineering Association for the latest general specifications for steel railroad bridges adopted by that Association. These specifications are used by a large percentage of the railroads in the United States.

Prob. 51. Find the maximum shear in an I beam 15 feet in span under the load represented in Fig. 87.

ART. 40. ANALYSIS OF A PLATE GIRDER.

To illustrate the method of determining the stresses when the live load consists of concentrated wheel loads, let a deck plate girder bridge for a single track railroad be taken, the span being 55 feet, measured between centers of bed plates, and the effective depth 5.5 feet. The total weight of both girders and of the lateral bracing is 16.45 tons, and the floor system is estimated at 420 pounds per linear foot. The live load is to consist of one Lehigh Valley consolidation locomotive and tender followed by a uniform train load of 4000 pounds per linear foot. It is required to find the maximum flange stresses and the maximum shears throughout the girder due to the above loads.

The total dead load for each girder is first found to be,

$$\frac{1}{2}\left(\frac{420 \times 55}{2000} + 16.45\right) = 14.0 \text{ tons,}$$

which is regarded as uniformly distributed. As this is a single track bridge the live load is divided by two, and for convenience the weights are reduced to tons.

On the axis AN, Plate III, and to a scale of 8 feet to an inch, are marked the positions of the wheel loads and the beginning, middle and end of a portion of the train 20 feet in length. Through these points indefinite verticals are drawn. On the load line hk, at the left of the plate, the wheel loads are laid off successively to a scale of 10 tons to an inch followed by 20 tons—the weight of the 20 linear feet of train. The pole o is chosen at a point above the middle of the load line, the pole distance being five times the depth of the girder, or 27.5 feet. It is not necessary to draw the rays from the pole, as the direction of each ray is determined by the pole and a point on the load line through which points the edge of the triangle is passed in the construction of the equilibrium polygon $A'B'FL'M'N'$. As the train load is not concentrated at its center but is uniformly distributed the required part $L'e'N'$ of the equilibrium polygon is a parabola tangent to $L'M'$ at L' and to $N'M'$ at N' (Art. 10). The construction is indicated on the diagram. The portion $A'B'$ of the polygon is a straight line parallel to ho and is produced as far as needed.

The left half of the girder is divided into five equal parts each 5.5 feet in length, and the sections numbered as shown. After erasing the lines of action of the wheel loads above the equilibrium polygon, a series of verticals are drawn 5.5 feet apart. The two verticals each marked *cc'* are 55 feet apart and *c'c'* is the closing side of the equilibrium polygon for the position *cc* of the girder. For this position the first driver stands at section 3 of the girder. The ordinate *Pd'* represents the flange stress at section 1 for this position of the load, the ordinate *Qe'* for section 2, and so on. The closing lines *a'a'* to *g'g'* are drawn, and all points on these lines distant one space from the left end are united by the curve *PP*, those distant two spaces from the left end by the curve *QQ*, those distant three spaces by *RR*, those distant four spaces by *SS*, and those distant five spaces by the curve *TT*. The ordinates between the polygon and the curve *PP* indicate the successive values of the flange stress at section 1 as the girder is moved from left to right with respect to the load, or, in other words, as the live load passes over the girder from right to left. The maximum ordinate between these lines is directly over the first driver, indicating that when the load is placed so that the first driver stands at section 1 it will give the maximum flange stress at that section. The maximum is always located at a vertex of the polygon, that is, at one of the loads. When the ordinate is not at an intersection through which the upper curve was drawn its length should be tested by drawing the closing line for the required position of the load. The pole distance *H* being five times the depth of the girder this ordinate must be measured with a scale five times that used on the load line or 50 tons to an inch. Applying the scale it is found to be 38.0 tons. The maximum ordinates to *QQ* and *RR* are 66.2 and 85.9 tons respectively, both being at the second driver; and the maximum ordinates to *SS* and *TT* are 97.0 and 99.6 tons, both being at the third driver.

The center of gravity of the locomotive and tender is 3 feet behind the third driver. When the load is so placed that the center of the girder is midway between these two points, only these loads being on the girder, the absolute maximum flange stress will occur in the section at the third driver. The section is therefore 1.5 feet from the center of the girder and the flange stress is 100.3 tons. The closing line for this position is shown near $c'c'$.

On an axis equal to the span in length and divided like the girder into ten parts the flange stresses just found are laid off as ordinates and a curve drawn through their upper extremities. Only one half is shown in the lower left side of the plate, the other half being symmetrical with it.

The diagram for flange stresses due to the dead load is now constructed below this axis by the method of Art. 35, and by measuring the ordinates the flange stress for the different sections of one-half of the girder are found to be 6.3, 11.2, 14.7, 16.8, and 17.5 tons respectively. These are to be added to the live load stresses in order to obtain the maximum stresses in the flanges.

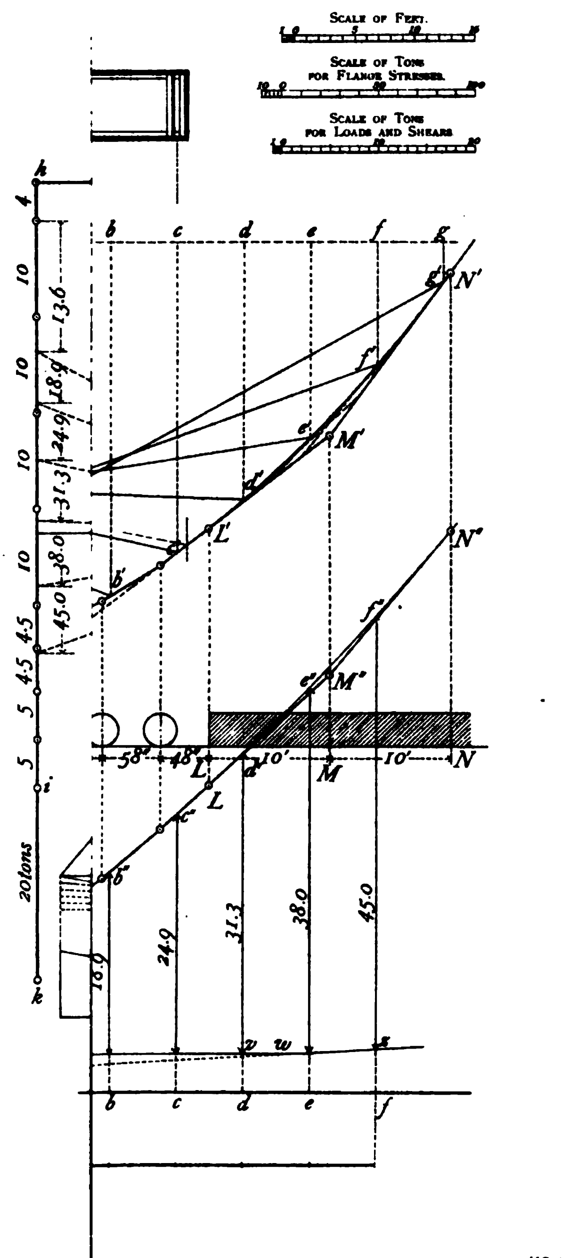
To determine the maximum live load shears another equilibrium polygon $A''B''D''L''N''$ is drawn by taking a new pole distance equal to the span, and as it is convenient to have the first side $A''B''$ horizontal the pole o' is placed directly opposite h, the beginning of the load line. From the first driver at C'' the span is laid off toward the right extending to f and then successive positions ee, dd, etc., of the girder are marked when it is moved toward the left 5.5 feet at a time. The ordinates $f''z$ . . $d''d$ . . $a''a$ are the corresponding reactions of the left support (Art. 37) and those portions of the ordinates above the line uwz are the maximum shears at the sections. The line uw is parallel to ae, ua equals 4 tons—the load on the pilot wheel—and wz is a part of $B''C''$ produced, w being 55 feet distant from B''. It is found that the maximum shear is caused in each section when the leading driver stands at that section and the load covers the right segment of the girder. For instance, when the first driver is at section 5 the shear is $a''u$ or 13.6 tons, when the pilot is placed at 5 the shear, measured on the ordinate at the left of $a''u$, is 10.6 tons, and for the second driver at 5 the corresponding ordinate (not shown on the plate) is still smaller. When the first driver is at the left support the reaction and also the shear at the support are equal to $f''z$ or 45.0 tons, $f''z$ lying between the polygon and the second side $B''C''$ produced, since the pilot is beyond the girder (Arts. 7 and 35). The shears for the sections from 0 to 5 are 45.0, 38.0, 31.3, 24.9, 18.9, and 13.6 tons respectively. If shears are desired for intermediate sections they may be measured directly on the diagram.

The above shears may also be obtained from the moment diagram used for finding the flange stresses. When the first driver is at section 5 the closing line of the polygon is $a'a'$ and, drawing parallel to this a ray through o, it is found to cut off on the load line a reaction of 17.6 tons or a shear of 13.6 tons. The other rays shown are parallel to the closing lines $b'b'$ to $f'f$. If one of the series of equidistant verticals did not coincide with the first driver another series of closing lines would have to be drawn to find the shears in this manner. The preceding method is usually the most economical in labor, but in this example the method just given has the advantage.

The shears due to dead load for the left half of the girder are given by the triangular shear diagram on the right of the lower end of the load line, and are to be added to the live load shears. Their values for the sections 0 to 5 are 7.0, 5.6, 4.2, 2.8, 1.4, and 0.0 tons.

Sometimes an approximate method is employed in which the pilot wheel is omitted from the system of loads. The differences between the true and the approximate shears in this

AIN LOADS.



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example are indicated on the diagram by the ordinates between the lines *uw* and $C''w$.

In practice the equidistant verticals corresponding to the divisions of the girder are drawn upon a separate sheet of tracing paper that may be shifted horizontally over the one on which the equilibrium polygons are constructed. This facilitates some parts of the process very materially.

If two coupled locomotives be specified instead of one the maximum flange stresses will be somewhat greater, as in that case the drivers of the second locomotive when placed at the various sections give the positions for maximum moment. This may be seen from the fact that the straight line $B'A'$ will be replaced by a broken line starting from B' and bending upward, thus raising the left ends of a number of closing lines and consequently increasing the ordinates between them and the polygon.

Prob 52. A plate girder for a single track bridge has a span of 31 feet 6 inches and a depth of four feet. Find the flange stresses and shears due to two coupled Pennsylvania consolidation locomotives.

ART. 41. ANALYSIS OF A PRATT TRUSS.

Let the example whose computations were made in Part I, Art. 63, be taken in order to compare the accuracy of the graphic method with the analytic method. The truss is a through Pratt for a double track railroad, of 140 feet span, having 7 panels, each 20 feet long and 24 feet deep. The dead load per linear foot is 1 400 pounds or 0.7 tons, and the live load a Pennsylvania typical passenger locomotive and tender followed by a uniform train load of 3 000 pounds or 1.5 tons per linear foot. Of the dead load 400 pounds per linear foot is to be taken on the upper chord.

As the construction required by this example (see Plate IV) is in many respects similar to that in the preceding article, only those features which are different need explanation. The linear scale used on the original drawing was 10 feet to an inch, and the scale of force 20 tons to an inch. The pole distance is taken as twice the depth of the truss or 48 feet. The verticals above the equilibrium polygon are 10 feet or half a panel length apart, and the maximum ordinates for the sections through the panel points are drawn in heavy lines. The ordinates are measured by double the scale used in laying off the load line or 40 tons to an inch, and their values marked on the diagram. The maximum ordinate below the curve *SS*, drawn for the section through the fourth panel point, is smaller than that for the third, and is the last one required for the chord stresses.

If the Pennsylvania consolidated locomotive be used instead of the passenger locomotive, the maximum ordinates for the sections at 3 and 4 will be found at such positions as to place the engine beyond the bridge, and toward the right of these ordinates the stress curves will be parallel to the equilibrium polygon.

In the shear diagram on Plate IV the ordinates at d'', b'', etc., represent the reactions of the left support for various positions of the load since the pole distance $o'h$ is 140 feet, the length of the span. For instance, the left reaction when the first driver is at 2 is the ordinate below b'' which measures (by the scale of 20 tons to an inch) 79.2 tons. The shear in the second panel is this reaction minus that portion of the load on both pilot wheels carried by the stringers to the floor beam at 1, or 9.4 tons. Laying this off from b'' downward and drawing a line parallel to the diagonal *CD* the stress for that member is found to be 90.8 tons. If any other load be placed at 2 a smaller stress is found.

In the fifth panel the positive live load shear is a maximum when the second pilot wheel is at the panel point on the right. The difference between the reaction and the shear is then 2.2 tons. The construction for two positions of the load is given for the fourth and fifth panels. The positive live load shear is a maximum in the sixth panel when the first pilot wheel is at panel point 6. The shear is then equal to the reaction, its value being 2.7 tons. The construction is omitted on the plate. The one shown is that due to the second pilot wheel at panel point 6.

The tension in the suspender BC is equal to that portion of the loads between 0 and 2 that is carried by the stringers to the floor beam at 1, such loads being brought on as to make it a maximum. This condition requires the drivers to be near 1. To determine this the small equilibrium polygon is drawn directly below the locomotive wheels using a pole distance equal to the panel length. The pole is o''. When the first six wheels are so placed between 0 and 2 that their center of gravity is at 1, the reactions at 0 and 2 are each 16.7 tons, as given by the ordinate above the intersection of the outer sides produced (Arts. 7 and 37). The reaction at 1 is therefore $(8 + 8 + 20 - 16.7)2 = 38.6$ tons. The first driver is next placed at 1, thus moving the second tender wheel beyond 2, and the sides of the polygon are produced to meet the vertical under this load. The reaction at 1 is then $64 - (9.4 + 15.0) = 39.6$ tons. When the second driver is placed at 1 the reaction at 1 is $64 - (9.6 + 14.8) = 39.6$ tons. The greatest live-load tension in BC is therefore 39.6 tons.

It will be observed by the student that the ordinate of 9.4 tons under the first driver gives the amount that is laid off on five of the ordinates on the large shear diagram, while the small ordinate of 2.2 tons under the second pilot is deducted from three of them.

Another method of finding the stress in BC when the first driver is at 1 is shown on the diagram including the load line, in which *ot* is parallel to the closing line *f'd'* and *ou* is parallel to the closing line *f'h'*.

The stress diagram for dead load when the diagonals all incline one way is shown on the left of the shear diagram.

The stresses thus obtained are for the chords,

	AD	*AF* and *AH*	*RB* and *RC*	*RE*	*RG*
Dead load........	− 58.3	− 70.0	+ 35.0	+ 58.3	+ 70.0
Live load.........	− 130.8	− 154.5	+ 81.8	+ 130.8	+ 153.5
Maximum.........	− 189.1	− 224.5	+ 116.8	+ 189.1	+ 223.5
Minimum.........	− 58.3	− 70.0	+ 35.0	+ 58.3	+ 70.0

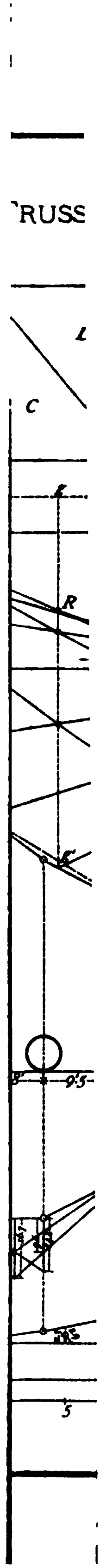
for the diagonals,

	AB	*CD*	*EF*	*GH*	*JK*	*LM*
Dead load.........	− 54.7	+ 36.5	+ 18.2	0	− 18.2	− 36.5
Live load on right..	− 127.8	+ 90.8	+ 59.4	+ 33.5	+ 15.2	+ 3.5
Live load on left...	0	− 3.5	− 15.2	− 33.5	− 59.4	− 90.8
Maximum.........	− 182.5	+ 127.3	+ 77.6	+ 33.5	0	0
Minimum..........	− 54.7	+ 33.0	+ 3.0	0	0	0

and for the verticals,

	BC	*DE*	*FG*	*HJ*	*KL*
Dead load........	+ 10.0	− 18.0	− 4.0	+ 10.0	+ 24.0
Live load.........	+ 39.6	− 45.6	− 25.8	− 11.7	− 2.7
Maximum.........	+ 49.6	− 63.6	− 29.8	......	......
Minimum.........	+ 10.0	− 6.3	− 4.0	......	......

The live load stress in *AH* is − 154.5, and that in *RG* is + 153.5 tons, because under the loads which cause both chord stresses of these magnitudes combined with the dead load the shear in the middle panel is negative, thus bringing into action the diagonal which slopes downward toward the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C
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left. As, however, the dead load shear is zero, its sign depends only on the live load and may therefore be most readily determined by drawing the corresponding closing lines on the moment diagram on Plate IV and observing the relative lengths of the ordinates at panel points 3 and 4, as explained in Art. 9. In both cases the ordinate at panel point 3 was found to be the greater. In practice the larger of the two values is often used for both chords of the middle panel.

The maximum and minimum stresses are found in the manner explained in Art. 30. The table shows that theoretically only the middle panel needs to be counterbraced, but practically counters would be inserted in the third and fifth panels to provide for some loads which might be allowed to cross the bridge before it would become necessary to replace the structure on account of the permanent increase in the weight of locomotives and trains demanded by the regular traffic.

As no counter tie is called into action in the third panel by the specified loads, the minimum stress in the vertical *DE* is shown to be $-18.0 + 11.7 = -6.3$ tons, because the live load stress in *DE* when the load comes from the left is equal in magnitude and opposite in sign to that in *HJ* (which equals the shear in *JK*) for the live load coming on the bridge from the right. Had the counter *JK* been required, then the minimum stress in *DE* would have been the dead panel load on the upper chord, or -4.0 tons. The stresses in *HJ* and *KL* given in the table are not real, as the adjacent counters, *JK* and *LM* never act under the given loads.

On comparing these results with those obtained by the analytic method it is seen that the average difference between them is about 0.1 ton, the only difference exceeding this amount being 0.3 ton for the chord members *AD* and *RE*, the value obtained by computation being 131.1 tons. The analytic method, however, gives another position of the live load which also satisfies the condition for the greatest stress in these chord members, namely, when the second driver stands at 2. The computed stress for the position is 130.3 tons. Two computations are also required for the stresses in $RB = RC$ and in GH.

Prob. 53. A through Pratt truss for a single track railroad bridge has 7 panels each 21 feet 6 inches long and 26 feet deep. The trusses are 14 feet 8 inches apart center to center of chords. The dead panel load is 1.93 tons on the upper and 6.33 tons on the lower chord. The live load is to consist of two coupled Lehigh Valley consolidation locomotives followed by a uniform load of 4 000 pounds per linear foot. Find the maximum and minimum stresses.

ART. 42. MOMENTS IN PLATE GIRDERS.

In Art. 40 the equilibrium polygon was drawn with a pole distance equal to a multiple of the depth of the girder, making it possible to read the values of the flange stresses directly from the diagram with the scale. This is convenient for a single example, but if a number of girders are to be analyzed for the same live load it will economize labor to plat the equilibrium polygon on a sheet of profile paper by laying off the moment ordinates under each wheel as taken from a tabulation diagram similar to the one shown in Part I, Art. 41. Such an equilibrium polygon, or wheel load moment diagram, is shown in the line $ABC'D'$ in Fig. 89. The first side AB is horizontal, and coincides with the axis AX. The total ordinate at any point represents the sum of the moments of all the wheels on the left of the point with reference to the point as a center of moments. If the side lying directly under the pilot (indicated by two sides of a triangle) of the second locomotive be produced to meet the ordinate at C', the portion of the ordinate above this line represents the moment of the entire second locomotive about the head of the train, while the portion below the side produced represents the moment of the first locomotive about the same

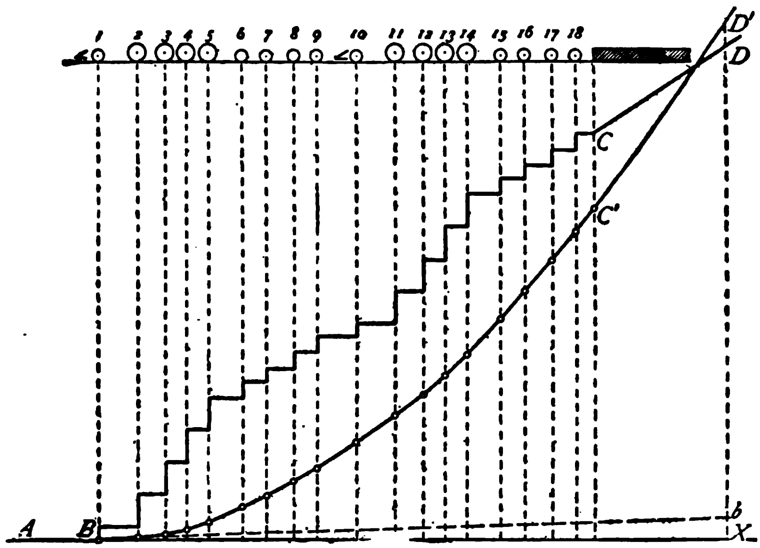


Fig. 89.

point, the values of these moments being read off directly from the sheet with its engraved scale (see Art. 7). As large a scale of moments should be adopted as possible without making the sheet too unwieldy to handle conveniently.

In the same article the maximum flange stress for any given section was obtained by practically finding the stresses for many different positions of the live load and selecting the largest one. If the position of the load which produces the maximum moment, and therefore the maximum flange stress, could first be determined, it would be necessary to find the value of only one moment for each section. This will now be done.

The criterion for the position of the wheel loads which produces the maximum moment at any section of a girder which supports the load directly, is

$$P' = \frac{l'}{l} W,$$

in which W is the whole load on the girder, P' the part of the load on the left of the section, l' the distance from the section to the left support, and l the span. This formula may be deduced in a similar manner to those in Part I, Art. 45. It is really the same formula as that which applies to vertical sections through the panel points of trusses with vertical posts, and to the panel points of the loaded chord of those with inclined web members. In plate girders with stringers and floor beams the criterion applies only to the sections at the floor beams. If the cross ties rest directly on the girder, or if a solid metal floor is adopted which is supported at points very close together like the cross ties, the load is regarded as supported directly, and the above formula may be applied at any vertical section whatever.

To use this criterion let a load line $ABCD$, composed of a series of steps, be constructed on the same sheet as the wheel load moment diagram, as shown in Fig. 89. The rise in any step indicates the weight and the position of the corresponding wheel. Any ordinate to this load line gives the sum of all the loads on its left, and its value can be read off directly.

On a sheet of tracing paper let the span ab of the girder be laid off to exactly the same linear scale as that of the profile paper containing the load line and moment diagram, and let the half span be divided into the required number of equal parts and indefinite ordinates erected at these points. Fig. 90 shows the completed diagram for a span of 80 feet placed in position on the load line. The live load used in this example is given in Fig. 87.

It is very important that when the base lines of the two diagrams coincide, their ordinates shall be truly parallel. The sections are five feet apart and the 20-foot section is made to coincide with wheel 4. If this be the proper position for

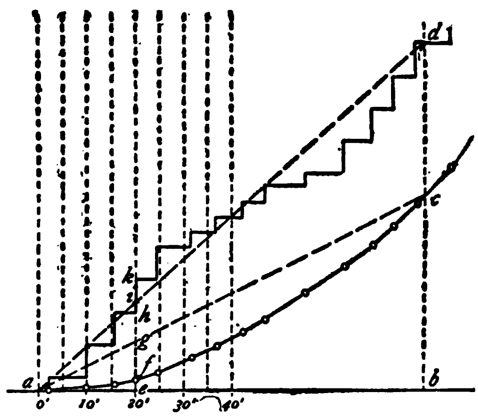


Fig. 90.

the maximum moment in the 20-foot section, the equation $P' = \frac{l'}{l} W$ must be satisfied. Remembering that the horizontal axis on the left of *a* is to be considered as a part of the load line, connect by a straight line the points *a* and *d* where the ordinates at the supports intersect the load line. Now the ordinate *bd* equals *W*, *ab* equals *l*, *ae* equals *l'*, and hence $W\frac{l'}{l}$ is represented by the ordinate *ei*. If the wheel 4 is just on the right of the section the ordinate *eh* represents *P'*, if just on the left the ordinate *ek* is the load *P'*; and when it is at the section the load *P'* may be regarded as having any value between these limits. The condition is therefore satisfied, and this position will give the maximum moment at section 20'. All the possible positions for all the sections can be determined in a few minutes, by using a small silk thread, shifting the tracing paper in each case so as to bring a wheel over the section, stretching the thread as indicated above and noting whether it intersects that wheel on the load line. The graphical method of determining position by means of a load line was first published by WARD BALDWIN in Engineering News, Vol. XXII, pages 295 and 345. See also letter of Dec. 11, 1889, on page 615.

By remembering the condition of loading required for uniform and excess loads, it is clear that the wheel loads should at first be approximately placed so as to cover the entire girder, and then if most of the drivers are on the left of the section the load should be moved somewhat toward the right. If by running off some light wheels on one end heavier ones may be brought on the girder at the other end without removing the drivers much farther from the section, the moment may probably be increased. By such considerations the labor of determining position may be reduced.

The value of the maximum bending moment at section 20′ may now be obtained by drawing the closing line and measuring the ordinate *fg*. A scale consisting of a separate strip of profile paper cut from the same sheet is convenient for this purpose. If the left end *a* of the closing line were always on the axis *ab*, greater precision would be attained by reading the moment *bc*, multiplying it by the ratio $l' \div l$, which in this example is one-fourth, and subtracting the moment *ef*, which is known and usually marked on the diagram. In plate girders, however, *a* is frequently not on the axis. In the above girder the positions satisfying the criterion for the various sections were found to be as follows:

Section.	Wheel at Section.	Section.	Wheel at Section.
5′	2, 3	25′	4, 12, 13
10′	2, 3, 4	30′	12, 13
15′	3, 4	35′	12, 13
20′	4, 12	40′	4, 5, 8, 9, 10, 12, 13

It is therefore better to make the scale large enough to insure the requisite precision when the ordinates are read off directly by a separate scale so that only one value needs to be recorded. Where more than one position satisfies the condition, the use of the dividers will show which is the largest, and its value alone needs to be carefully read and recorded. In order to enable a large vertical scale to be used on the width of an ordinary sheet of profile paper, the equilibrium polygon, or moment diagram, was consolidated as shown in Fig. 91. In this way the vertical divisions on profile paper, plate B, which measure a little more than 0.8 inch, could be taken as 500 thousand pound-feet instead of 1000 thousand pound-feet

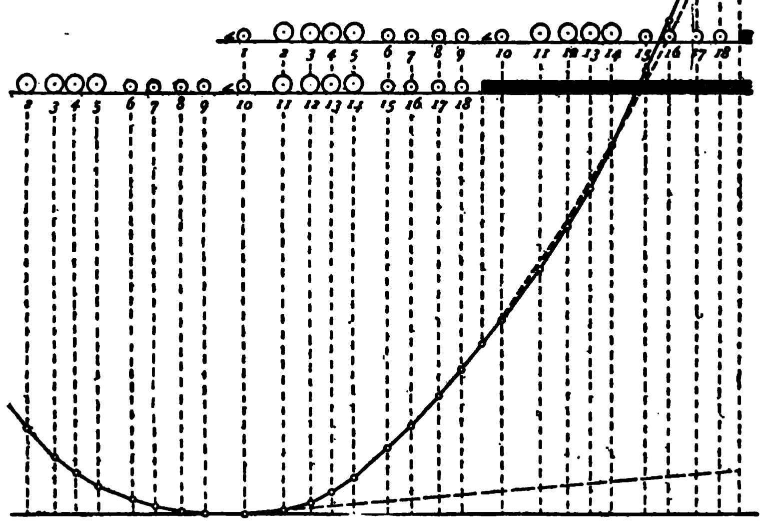


Fig. 91.

as before. The linear scale used was 4 feet to an inch, the actual working diagrams being therefore about ten times as large as those here given.

By reference to the above table of positions it is seen that the ordinates to be measured lie in the left half of the diagram so that acute intersections of the ordinates with the polygon are avoided. If this diagram were not also to be used for obtaining shears, a further improvement would be made by inclining the axis downward towards the right, thus bringing the closing lines more nearly horizontal.

Referring again to Fig. 90 in order to call attention to the relation between the analytic and graphic methods, it should be observed that if no load is off the girder at the left, the ordinate *eg* is the moment of the reaction while *ef* is the moment of the loads on the left of the section, the center of moments being at some point in the section. If, however, some loads have passed beyond the left support, then the corresponding point *e* will lie on that side of the equilibrium polygon produced which is intersected by the vertical at the left support.

In plotting the load line and the equilibrium polygon the values generally used in practice correspond to the weights on one rail only, since single track bridges are so much more numerous than those with a double track. As it is customary to mark the stresses in pounds on stress sheets, it is preferable to plat the weights in units of 1000 pounds and the moments in units of 1000 pound-feet.

If a plate girder has floor beams and stringers, its bending moment diagram for the live load, corresponding to that shown above the axis in the lower left corner of Plate III, will be bounded by irregular curves between the moment ordinates at the floor beams, or panel points. These curves may be concave as well as convex, and may be replaced by right lines for all practical purposes. The more precise determination of intermediate moments will be treated in Art. 56.

An accurate representation of the dead load bending moment diagram consists of two parts, one due to the weight of the floor system, which is concentrated at the panel points and which may be plotted above the axis; the other due to the uniformly distributed weight of the girder itself. Fig.

92 shows the moment diagrams of a through plate girder whose span is 80 feet, and whose floor system has five panels.

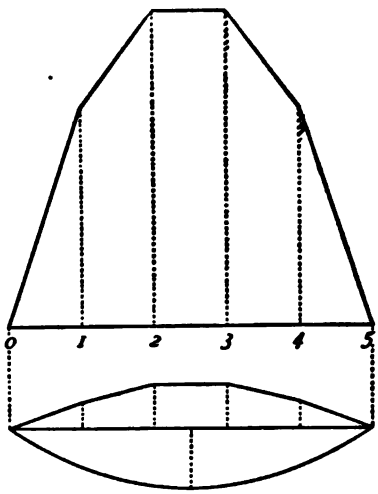


Fig. 92.

The upper diagram gives the moments due to live load, and the lower one those due to the dead load.

Prob. 54. Construct the diagrams described in this article, check the positions given in the table, find which position gives the maximum moment at each section and the values of these moments.

ART. 43. SHEARS IN PLATE GIRDERS.

In Art. 40 it was shown that the maximum shear in any section occurs when one of the wheels at the head of the locomotive is at the section. If M' be the moment of all the loads on the girder about the right support when wheel 1 is at the section and M'' when wheel 2 is at the section, the corresponding values of the vertical shear are

$$V_1 = \frac{M'}{l}, \quad \text{and} \quad V_2 = \frac{M''}{l} - P_1,$$

P_1 being the weight of wheel 1. If the load is directly supported by the girder there will be some section where the shear due to both positions is equal. Equating these values and transposing,

$$M'' - M' = P_1 l.$$

In Fig. 93 let cd be the moment M'' and ab the moment M'. The distance ac between these moment ordinates is

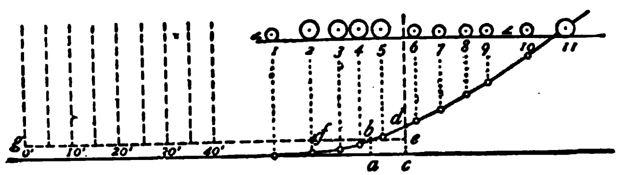


Fig. 93.

equal to the distance between wheels 1 and 2. If $V_1 = V_2$, $de = cd - ab = M'' - M' = P_1 l$. The position of the section where $V_1 = V_2$ may then be found as follows: Place the girder diagram (constructed on the tracing paper as described in the preceding Article) on the locomotive moment diagram (Fig. 89) with its left support at wheel 1, and mark on its section or ordinate at the right support the distance to the line Bb, which is the side of the equilibrium polygon on the right of wheel 1 produced. This is $P_1 l$ (Art. 7). Move the former diagram to the left until the section at the right support is at wheel 2, and mark the position of wheel 1. The two points marked are shown in Fig. 93, at d and b, respectively. Now move the tracing with the point d remaining on the equilibrium polygon and with its axis horizontal until a position is reached where b is also on the polygon. Mark the position of wheel 2 at f. It is easily remembered what wheel is to be marked by noticing that the right-hand ordinate cd is M'', which by the notation is the moment at the support when wheel 2 is at the section. At every section, therefore, between f and e the greatest shear will be produced when wheel 1 is at the section, and for all sections between f and g the left support) when wheel 2 is at the section.

In the girder under consideration the shears under the first two wheels are equal at a section a little over 60 feet from the left support. For the live load here employed no plate girder has been built of so large a span as to cause wheel 3 at any section to give a greater shear than wheel 2.

The load is now placed in position for the different sections successively, and the corresponding moments at the right support read off. When these are divided by the span the left reactions are obtained, and from the reactions the shears are found by subtracting P_1 in the case of those sections for whose position P_1 lies between the section and the left support. For the section at 15′ the moment at the right support is 7 040 thousand pound-feet. This gives a reaction of 7 040 $\div$ 80 = 88.0, and a shear of 88.0 − 8 = 80.0 thousand pounds, or 80 000 pounds. For the section 0 wheel 1 is off the girder, and hence the ordinate below the line *Bb* in Fig. 89 must be deducted from the moment at the right support. The shear at 0 is found to be (10 225 − 705) $\div$ 80 = 119.0 thousand pounds. The values of the shears from sections 0 to 40′ inclusive are 119.0, 105.6, 92.6, 80.0, 68.8, 58.9, 50.1, 41.8, and 33.9, all expressed in units of a thousand pounds. The form of diagram employed (Fig. 91) indicated at a glance that a single locomotive followed by a train produces slightly greater shears at sections 15′, 20′, 25′, and 30′ than the load with two locomotives, their values being 80.4, 69.8, 59.8, and 50.4 thousand pounds respectively.

Usually only the maximum shears in one-half of the girder are required, but in order to show the variation of shear across the entire girder as well as the minimum values, the shear diagram due to dead and live load is shown in Fig. 94. The greatest live load shears (all positive) are laid off above the axis and the dead load positive shears below the axis, so that the diagram combines their values. The point of zero shear is at 55.6′ or at 15.6 feet from the middle of the girder.

The shear may therefore change sign at all points within this distance on each side of the middle. If the live load were uniformly distributed, the shear curve would be a parabola, with its vertex at the right end of the girder.

When the live load consists of passenger instead of consolidation locomotives it is possible that for some spans the section in which $V_2 = V_3$ may be on the right of that where $V_1 = V_2$, in which event it is necessary to find where $V_1 = V_3$. No position for maximum shear then requires wheel 2 to be at any section.

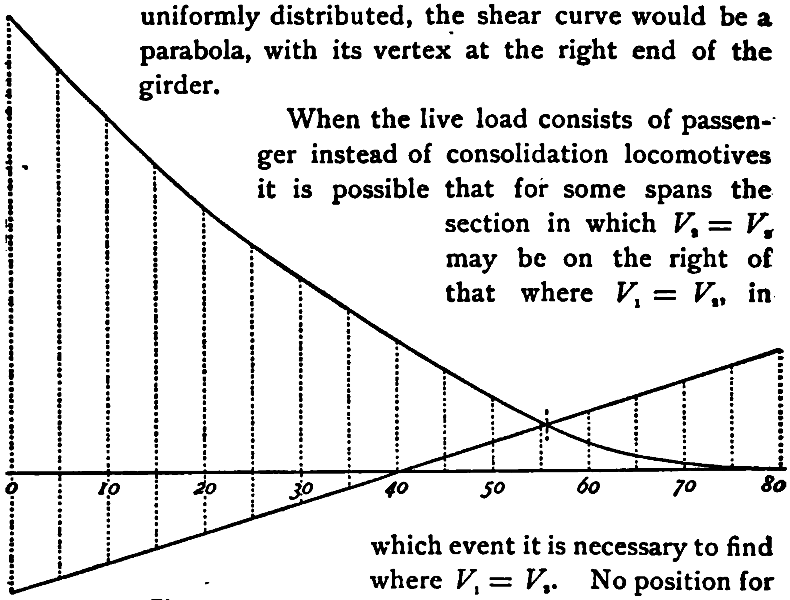


Fig. 94.

If the girder is divided into panels by floor beams the criterion for position for maximum shear is the same as for trusses with parallel chords. The necessary formula was deduced in Part I, Art. 43, but its graphic application will be deferred to Art. 45. The live load shear curve would be transformed into a series of steps like that in Fig. 81, and the dead load shear diagram would consist of two parts, one for the floor system which is concentrated at the panel points, and the other for the weight of the girder itself, which is uniformly distributed.

Prob. 55. Take the plate girder used in the example in Art. 40, find where $V_1 = V_2$, and the values of the maximum shears due to one Lehigh Valley consolidation locomotive and train.

ART. 44. SIMULTANEOUS MOMENTS.

In designing the riveting of the flanges to the web of a plate girder it is necessary to have the horizontal shear between them or the increments of flange stress between the sections for which the bending moments and vertical shears were found. If the sections are a distance dx apart the difference of bending moments dM is a maximum when the load is so placed as to cause the vertical shear to be a maximum, since from mechanics $dM = Vdx$. When the distance between the sections is greater than dx the difference of bending moments is a maximum when the loads are so placed that the vertical shear is a maximum in the section nearer the middle of the span, and this holds true for uniform loads until the sections are separated a distance somewhat less than half the span. As the sections are not taken farther apart than the depth, which even in short spans is generally less than one-eighth of the span, the exact value of the limiting distance referred to need not be determined.

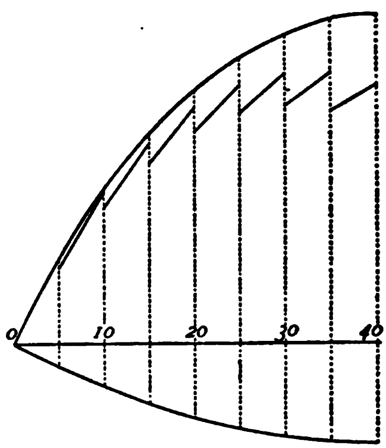


Fig. 95.

The following table contains the simultaneous bending moments in each pair of adjacent sections of the girder used in the preceding two Articles when the live load is so placed as to produce the maximum shear in the section nearer the middle of the girder. The moments are expressed in thousand pound-feet. The differences in this table are to be increased by the corresponding differences in the dead load bending moments. The moments themselves are laid off as ordinates in Fig. 95, those of each pair being joined by a right line. The upper curve gives the maximum live load bending

Distance of Section from Support.	Load in Position for Maximum Shear at Section.								Distance of Section from Support.
	5′	10′	15′	20′	25′	30′	35′	40′	
Feet.									Feet.
0	0	....	......	......	1315	......	......	.....	20
5	530	480	......	......	1610	1430	......	.....	25
10	...	945	850	......	......	1680	1470	.....	30
15	....	....	1255	1125	......	......	1680	1445	35
20	....	....	......	1470	......	......	......	1610	40
Difference	530	465	405	345	295	250	210	165	Difference

moments. Those due to dead load are laid off below the axis.

Prob. 56. Find the simultaneous moments in the sections of the plate girder used in the example in Art. 40 (see Plate III).

Art. 45. Shears in Trusses.

In a similar manner to that explained in Arts. 43 and 42 for a plate girder the shears and bending moments in the Pratt truss in Art. 41 can be found by means of a single diagram for the locomotive and train load, the positions which give the maximum stresses being first determined by means of the load line.

In Part I, Art. 43, the criterion for the position of the live load producing the greatest vertical shear in any section of a truss was found to be $P'' = \frac{1}{m}W$, in which W is the whole load on the truss, P'' the wheel loads (one or more) on the panel cut by the section, and m the number of panels in the truss. Let this equation be transformed into $W = mP''$. When the truss diagram is placed on the load line the value of W can at once be read off on the ordinate at the right support for any given position of the truss with respect to the loads. If wheel 2 be placed just on the right of a panel point W must be equal to mP_1, and if just a little to the left of the same panel point then W must be $m(P_1 + P_2)$. The condition will therefore be satisfied if wheel 2 is at the panel point, and the value of W is found to lie between the values mP_1 and $m(P_1 + P_2)$, and similarly for any other wheel.

In this example, $m = 7$, $P_1 = 8$ tons, $P_1 + P_2 = 16$ tons, and $P_1 + P_2 + P_3 = 36$ tons. The following table is then arranged, like that shown in Part I, Art. 43, for the corresponding analytic method.

No. of Wheel at Right End of Panel.	Values of P''.	Corresponding Values of mP''.
1	0– 8 tons.	0– 56 tons
2	8–16 "	56–112 "
3	16–36 "	112–252 "

The moment diagram of the single Pennsylvania passenger locomotive and train is constructed like that illustrated in Fig. 89, and the truss diagram (like Fig. 96) is drawn to the same linear scale on a sheet of tracing paper, but with the verticals extended as high as the moment diagram. To find

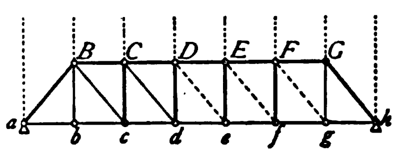


Fig. 96.

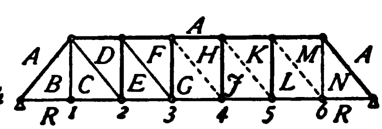


Fig. 97.

the position for maximum shear in Cc and Cd due to this load shift the truss diagram on that of the load line until wheel 3 is at d. If this is the correct position the load on the truss must be between 112 and 252 tons. This is found to be the case, its value being 154 tons. For the greatest positive shear in Fg (which equals the greatest negative shear in Bc) wheel 2 is placed at g. The load W is then 56 tons, which just meets the condition and indicates that wheel 1 at *g* should also be tried. In the latter case *W* is 36 tons. The positions are all recorded in the following table. In practice it is preferable to obtain all the positions before reading off any moments from the equilibrium polygon, that is, to deal with only the load line at first and with the equilibrium polygon afterwards.

PANEL.	WHEEL AT RIGHT END OF PANEL.	MOMENT AT RIGHT SUPPORT.	MOMENT AT RIGHT END OF PANEL.	SHEAR.	STRESS.	WEB MEMBER..
ab	3	15075	188	98.3	− 128.0	*aB*
bc	3	11100	188	69.9	+ 91.0	*Bc*
cd	3	7730	188	45.8	− 45.8	*Cc*
					+ 59.6	*Cd*
de	3	4940	188	25.9	− 25.9	*Dd*
					+ 33.7	*De*
	2	3885	44	25.5		
ef	2	1965	44	11.8	+ 15.4	*Ef*
fg	2	645	44	2.4		
	1	390	0	2.8	+ 3.6	*Fg*

The moments in the third column of the table may now be read directly from the diagram. Those in the next column were computed and marked on the diagram when it was constructed. They are all expressed in tons-feet. Since the span is 140 feet and the panel length 20 feet, the shear in *Cc* and *Cd* is $(7\,730 \div 140) - (188 \div 20) = 45.8$ tons. As it is not necessary to know the value of the reaction separately, and as usually the panel length is not such a simple number as the above, it is better to multiply the moments in the fourth column by the number of panels in the truss, subtract the products from the moments in the third column, and divide the remainder by the span. Thus the shear in *Cc* and *Cd* is $(7\,730 - 7 \times 188) \div 140 = 45.8$ tons. The stresses are obtained from the shears either graphically or by the aid of logarithms, as may be more expeditious.

As the method described above is the exact graphic equivalent of the analytic method given in Part I, Art. 44, the student should make a careful comparison between them. Fig. 97 is placed beside Fig. 96 as an aid in comparing the results obtained by the analytic and the two graphic methods.

The criterion for position used in this Article applies to all trusses with horizontal chords and single systems of webbing, like the Howe and the Warren as well as the Pratt truss. When one or both of the chords are inclined, the maximum shear does not give the maximum web stresses, since the chord takes some of the shear. The method of finding the position of the live load for trusses with inclined chord members will be given in Art. 49.

Prob. 57. The Pratt trusses of a single-track through railroad bridge have seven panels, each 25 feet 8 inches long and 32 feet deep. Find the shears and stresses in all the web members except the suspenders due to two Lehigh Valley consolidation locomotives and train (see Fig. 87 in Art. 39).

ART. 46. FLOOR-BEAM REACTIONS.

In order to find the maximum floor-beam reaction or stress in the suspender Bb due to locomotive wheel loads it is necessary to deduce additional formulas. In Fig. 96 let R_a be the stringer reaction at a, and R_b the sum of the adjacent stringer reactions, or the floor-beam reaction, at b. Let P be the whole load on the two stringers of equal spans ab and bc, and g the distance of the center of gravity from c; let P' be the load on ab, and g' the distance of its center of gravity from b. Since the sum of the moments of loads and reactions about b is zero,

$$R_a p - P'g' = 0, \quad \text{or} \quad R_a p = P'g'.$$

Taking moments about c,

$$R_a 2p + R_b p - Pg = 0.$$

Substituting and reducing,

$$R_b = \frac{Pg - 2P'g'}{p}.$$

If the loads be moved a distance dx to the left both g and g' will receive an increment dx, and R_b an increment

$$dR_b = \frac{Pdx - 2P'dx}{p}.$$

Placing the derivative equal to zero gives

$$P = 2P'.$$

That is, when the live load in both panels is double that in the panel ab the resulting value of R_b is a maximum. This is the same condition as for finding the maximum bending moment at the middle of the girder whose span is ac.

The use of the load line gives the position very quickly, and the moments Pg and $P'g'$ can be read off on the moment diagram. If M_c be the moment ordinate at c, and M_b that at b, the value of R may be more conveniently expressed and remembered as

$$R_b = \frac{M_c - 2M_b}{p}.$$

In the case of bridges where the two panels at the end, p_1 and p_2, are not equal, the value of R_b deduced in a similar manner is

$$R_b = \frac{p_1 M_c - (p_1 + p_2)M_b}{p_1 p_2},$$

the criterion for loading being that which produces a maximum moment at b in a girder whose span is ac, p_1 being the span of the stringer ab, and p_2 the length bc.

In applying this condition for loading, one of the heaviest loads should be placed at b and as large a load brought on the two panels from a to c as possible. When wheel 3, in the example used in the previous Article, is placed at b and the thread is stretched to unite the intersections of the ordinates at *a* and *c* with the load line, it crosses the step representing wheel 3 and hence satisfies the condition. The moment at *c* is 1 170, and that at *b* is 188 tons-feet, whence $Bb = (1\ 170 - 2 \times 188) \div 20 = 39.7$ tons. When wheel 4 is at *b* the condition is also satisfied, and the corresponding stress found to be the same as for the other position.

Prob. 58. Find the stresses in the suspenders in Prob. 57 in the preceding Article.

ART. 47. MOMENTS IN TRUSSES.

The method used in the preceding Article in obtaining the maximum shears will now be employed to find the maximum chord stresses in the same truss and due to the same wheel loads.

The condition for loading is expressed by a formula deduced in Part I, Art. 45, which, in slightly modified form, was given in Art. 42 and its application to a plate girder fully explained. As there stated, it applies only to the bending moments in vertical sections through the panel points of the loaded chords of trusses. The positions are recorded in the following table for the required sections of the left half of the

SECTION.	WHEEL AT SECTION.	MOMENT AT RIGHT SUPPORT.	MOMENT AT SECTION.	BENDING MOMENT.	STRESS.	CHORD MEMBER.
Bb	3	15075	188	1966	81.9	$ab = bc$
	4	16850 − 1140	476 − 180	1948		
Cc	5	14540	1008	3146	131.1	$cd = BC$
	4	12610	476	3127		
Dd	8	13610	2124	3709	154.5	$CD = DE$
Ee	train	14510	4605	3686	153.6	*de*

truss. As for the section *Ee* the train is at the section the direction of the thread must coincide with the straight portion of the load line representing the train, and hence the left end of the truss must be placed at the point where the line produced meets the horizontal axis.

For a truss having equal panels, as in this example, time may be saved as well as increased precision secured by not drawing the closing lines and reading the bending moments directly, but by reading the moments at the right support and at the given section. The latter, being at a wheel, has its value marked on the diagram, and is hence quickly obtained. The subtractive moments for the second position for section *Bb* are those due to wheel 1, which is off the bridge.

The computation for the bending moment is very simple for trusses with equal panels, as it is not necessary to obtain the value of the reaction separately. For wheel 5 at *c* the bending moment in the section *Cc* is

$$\frac{2}{7}(14\,540) - 1\,008 = 3\,146 \text{ tons-feet,}$$

and when divided by the depth the chord stress is $3\,146 \div 24 = 131.1$ tons.

Since there is no dead load shear in the middle panel, it is necessary to find which one of the diagonals is acting for each of the positions for sections *Dd* and *Ee*. The shear equals the left reaction of the truss minus the loads from *a* to *d*, minus the reaction at *d* of the stringer *de*. By producing the side of the equilibrium polygon immediately on the left of *d* when wheel 8 is at *d* and reading the moment intercepted above this line at *e* the moment of the stringer reaction at *d* about *e* as a center is obtained. Its value is found to be 380 tons-feet. The shear is therefore

$$\frac{13\,610 - 7 \times 380}{140} - 80 = -1.8 \text{ tons.}$$

As the diagonals can take only tension, this shear calls *dE* into action, and hence the bending moment for the section

Dd gives the chord stress in both *CD* and *DE*. For the last position the entire panel is covered by the train load, and therefore the stringer reaction at *d* is 15 tons. The shear is

$$\frac{14510}{140} - 90 - 15 = -1.4 \text{ tons.}$$

Since this also stresses the diagonal *dE*, the moment for the section *Ee* gives the chord stress in *de*.

The sign of the shear without its magnitude may be more quickly determined for each of these positions by drawing the closing line of the equilibrium polygon and with the aid of the dividers finding whether the bending moment at *d* is greater or less than the simultaneous moment at *e*. In the former case the shear is negative and in the latter positive (see Art. 9).

If the live loading for the greatest moments at the sections *Dd* and *Ee* respectively had caused shears of opposite signs in the middle panel, the required stress in one of the chords of that panel would not have been given by either loading. In such a case it becomes necessary to shift the load to some intermediate position for which the bending moment ordinates at both ends of the middle panel are equal, and the shear is therefore zero. The required stress may then be obtained from the moment at either section. In practice, however, the stresses in both chords of the middle panel are generally assumed to be equal.

When the center of moments of a chord member is situated on the unloaded chord of a truss, and does not lie in the vertical section through a panel point of the loaded chord, either formula (1) or (2) given in Part I, Art. 45, determines the condition of loading. An example of their application will be given in Art. 56. The condition in all cases applies to inclined as well as to horizontal chords.

Prob. 59. Find the greatest live load stresses in the chords in Prob. 57 in Art. 45.

CHAPTER V.

TRUSSES WITH BROKEN CHORDS.

ART. 48. POINTS OF DIVISION IN PANELS.

For the truss in Fig. 98 the position of the locomotive wheel loads, or of any other class of live load which produces the greatest stress in any chord member, is found by the same criterion as if the chords were both horizontal. The same

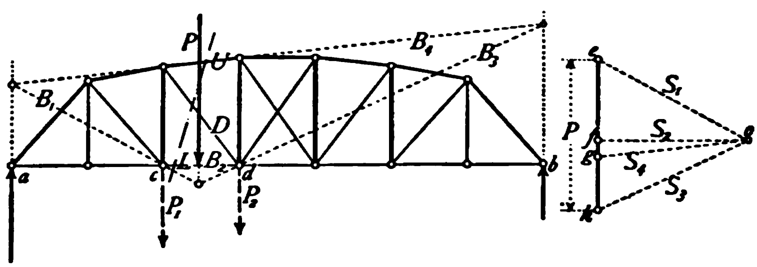


Fig. 98.

statement would be true if both chords were broken or curved. On the other hand, the stress in any web member for the position of the live load which causes the maximum shear in the section is not the greatest in this case, because the inclination of any chord member cut by the section causes it to take some of the shear. It will be necessary, therefore, to find the condition of loading which is required by trusses with inclined chord members.

The section shown in Fig. 98 cuts the upper chord member U, the diagonal D, and the lower chord member L. By the method of moments the center of moments for D is at the intersection of U and L, some distance beyond the figure on the left. If a single concentrated load P_1 be placed at c or at any point on the left of c, it will cause compression in the diagonal D. This is readily seen to be the case, since the stress in D (which is directed away from the section, and hence downward) holds in equilibrium the forces on the left of the section, and therefore their resultant. The resultant of P_1 and the reaction at a is a downward force at the right support b, and hence its moment is positive. If a single load P_2 be placed at d or at any point on the right of d, it will then cause tension in the diagonal, for the stress in D holds in equilibrium only the upward reaction at a whose moment is negative.

When a load P is placed between c and d, the floor system of the bridge transfers a portion P_1 to the panel point c of the truss and the remaining portion P_2 to the panel point d, that is, the load P is replaced on the truss by its components P_1 and P_2. As one of these causes compression and the other tension in D, there must be some position of the load P for which the resulting stress in D is zero.

Let U be produced to meet the verticals at a and b, let these points of intersection be joined with c and d, and the lines produced until they meet. Let P be placed directly over this latter intersection. On the right of the figure is shown a force polygon, in which eh is laid off by scale equal to P, and the rays S drawn parallel to the corresponding lines B as indicated by their subscripts. The lines B_1, B_3, and B_4 form an equilibrium polygon, and hence eg equals the reaction at a, and gh the reaction at b. Portions of the lines B_1 and B_4 and the line cd, or B_2, form another equilibrium polygon, and therefore ef represents P_1 and fh represents P_2. Consequently B_1, B_2, B_3, and B_4 form an equilibrium polygon for the loads P_1, P_2, and the reactions at a and b respectively. The line of action of the resultant of P_1 and the reaction at a must pass through the intersection of B_4 and B_3 (Art. 7), which by construction coincides with the center of moments. The moment of the resultant is therefore zero, whence it follows that the stress in D is also zero.

The position of a concentrated load P causing no stress in any web member can therefore be found by the following rule:

> Pass a section cutting the web member and a member of each of the chords. Produce the unloaded chord member to an intersection with the verticals at the supports. Join these points with the panel points at the end of the loaded chord member. The intersection of these lines gives the required position.

From the manner in which the above investigation was made it is clear that this rule applies also to a truss in which both chords are curved, and for webbing whose posts are not vertical. The rule is therefore stated in its general form and applies to deck as well as through bridges.

The manner in which the section must be cut to obtain the stresses in the vertical posts depends upon which diagonals are acting in the adjacent counter-braced panels (see Part I, Art. 36, and Part II, Arts. 30 and 35). The position of P which produces no stress in the vertical on the left of D in Fig. 98 when both of the adjacent main diagonals are acting is somewhat nearer to panel point d.

The results of this investigation also show that if the live load consists of panel loads, all the panel points on the right of P are to be loaded for the greatest tension in D. If one excess load is employed it must be placed at d. For the greatest tension in the counter diagonal in the same panel the load is similarly placed at c, and the panel points on the left. This loading, it will be observed, does not differ from the corresponding one for horizontal chords.

If the live load is uniformly distributed it must extend from the right support to the position of P for the greatest tension in D, and from the left support to the position of P for the greatest tension in the counter diagonal. When the construction shown in Fig. 98 is applied to trusses with horizontal chords it gives the positions for true live load shear which were determined by the analytic method in Part I, Art. 43. The position of locomotive wheel loads which produces the greatest stress in D will be found in the next article.

Prob. 60. Find the position of P for all the diagonals and the second, third, and fourth verticals of the bowstring truss in Fig. 74, Art. 35.

ART. 49. POSITION OF WHEEL LOADS.

In Fig. 99 the position of P in Fig. 98 which causes a stress of zero in the diagonal is indicated by the vertical marked z. Let the stresses in the diagonal, lower chord, and upper chord, cut by a section through the panel cd be denoted by

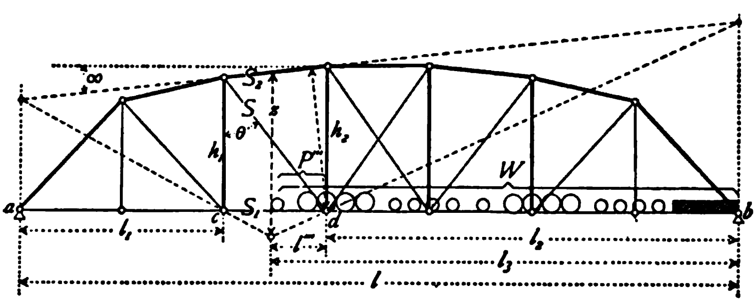


Fig. 99.

S, S_1, and S_2 respectively, and the depths of the truss at c and d by h_1 and h_2. Let the total weight of the wheels (one or more) on the panel cd be P''', and the distance of its center of gravity from d be g''', W being the weight of the entire load on the truss, and g the distance of its center of gravity from the right support. Let the bending moments at the upper and lower extremities of D be M_1 and M_2 respectively. The remaining terms employed in the following discussion are shown in the figure.

Let the segment of the truss on the left of the section cutting S_2, S, and S_1 be considered. Resolving horizontally,

$$S_2 \cos \alpha + S \sin \theta + S_1 = 0.$$

The lever arm of S_2 makes the same angle α with the vertical h_2 as S_2 makes with the horizontal. Taking moments about d,

$$M_2 + S_2 h_2 \cos \alpha = 0, \quad \text{whence} \quad S_2 \cos \alpha = - M_2 \div h_2;$$

and taking moments about the panel point at the upper end of D,

$$M_1 - S_1 h_1 = 0, \quad \text{whence} \quad S_1 = M_1 \div h_1.$$

After substituting these values above, there follows,

$$-\frac{M_2}{h_2} + \frac{M_1}{h_1} + S \sin \theta = 0, \quad \text{or} \quad S \sin \theta = \frac{M_2}{h_2} - \frac{M_1}{h_1}.$$

The last equation is an important one, and indicates that the horizontal component of the stress in any web member equals the difference of the quotients obtained by dividing the bending moments at the extremities of the member by the corresponding depths of the truss at those points.

The values of the bending moments are

$$M_1 = \frac{Wgl_1}{l} \quad \text{and} \quad M_2 = \frac{Wg}{l}(l - l_2) - P'''g'''.$$

From similar triangles

$$h_1 : s = l_1 : l - l_2 \quad \text{and} \quad h_2 : s = l_2 : l_3;$$

whence

$$h_1 = \frac{l_1 s}{l - l_2} \quad \text{and} \quad h_2 = \frac{l_2 s}{l_3}.$$

Substituting these values of M_1, M_2, h_1, and h_2, reducing, and finally replacing $(l_2 - l_1)$ by l''',

$$S = \frac{Wgl''' - P'''g'''l_2}{l_2 z \sin\theta}.$$

If the load advance a distance dx, both g and g''' receive an increment of dx, and the stress S receives an increment of

$$dS = \frac{(Wl''' - P'''l_2)dx}{l_2 z \sin\theta}.$$

Placing the derivative equal to zero gives the condition which makes S a maximum, which is $Wl''' - P'''l_2 = 0$, or, when put into more convenient form for use,

$$P''' = \frac{Wl'''}{l_2}.$$

This formula is very convenient to use graphically, and as it is similar in form to that for maximum moment (Arts. 42 and 47) it is to be treated in like manner. Referring to Fig. 100, which illustrates the truss diagram (drawn on tracing paper) placed in position on the live load moment diagram, bg represents the

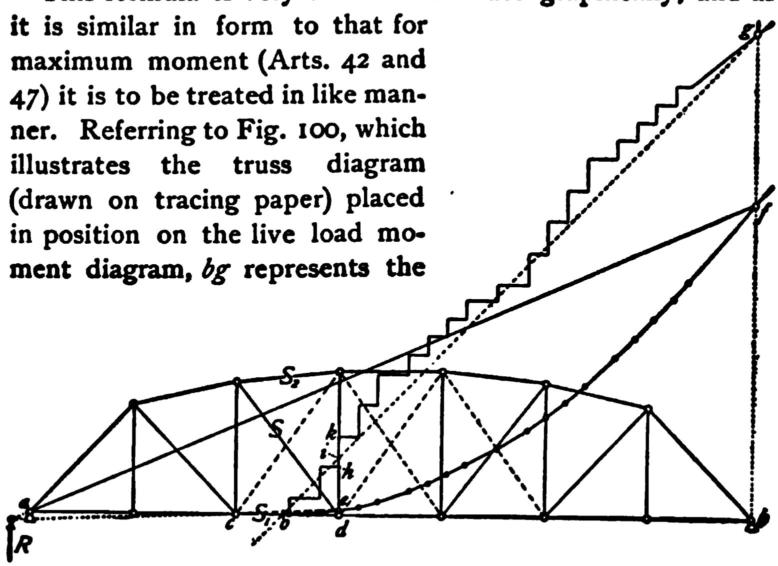


Fig. 100.

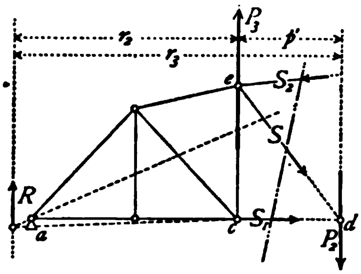
total load W, ob the distance l_1, and od the distance l'''. The ordinate di is therefore equal to $Wl''' \div l_1$, and this must equal P''' if the position is correct. When the load is so placed that a wheel is just on the right of the panel point d, the load P''' is represented by dh, and if just to the left of it by dk; hence if i lies anywhere between h and k, or, in other words, if the thread stretched from o to g cuts the load placed at the panel point, the criterion for position is satisfied. A wheel must always be placed at the panel point, and while usually the first wheel is at the right of o, it may sometimes happen that the condition is met when the first wheel is a little to the left of o. After the right position is found the moment ordinates bf and de are read off as usual.

Prob. 61. A double track through railroad bridge has trusses of the type illustrated in Fig. 100. There are 12 panels, each 30 feet long. The depths at panel points 1 to 6 inclusive are 29′ 0″, 41′ 0″, 49′ 5″, 53′ 4″, 58′ 10″, and 60′ 0″ respectively. The live load is WADDELL'S Compromise Standard, Class U (Art. 39). Find the position of the live load which shall produce the greatest stresses in the main and counter ties and the posts. Six panels require counter bracing. Also compare these positions with what they would be if the truss had parallel chords.

ART. 50. RESOLUTION OF THE SHEAR.

In Fig. 100 in the preceding article the stresses S_2, S, and S_1 hold in equilibrium the external forces on the left of the section cutting these members. These external forces consist of an upward reaction at a and a downward force at c equal and opposite to the left reaction of the stringer in the panel cd. The resultant R of these two forces is an upward force whose line of action is a little to the left of the support a. Its position may be readily determined, if desired, by drawing the closing line af and the chord ce of the equilibrium polygon and producing them to their intersection.

Referring now to Fig. 101, let the resultant R be replaced by two forces P_2 and P_3, the former acting downward at panel



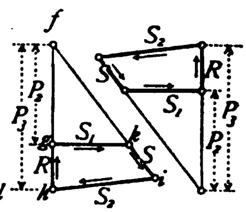


Fig. 101. Fig. 102. Fig. 103.

point d and the latter acting upward at e. The points d and e are at the extremities of the diagonal cut by the section. Taking moments about e, and remembering that the bending moment at e is M_2,

$$Rr_2 = P_2p', \quad \text{and} \quad P_2 = \frac{Rr_2}{p'} = \frac{M_2}{p'}.$$

Similarly taking moments about d,

$$Rr_3 = P_3p', \quad \text{and} \quad P_3 = \frac{Rr_3}{p'} = \frac{M_3}{p'}.$$

Taking vertical components,

$$R = P_3 - P_2 = \frac{M_3}{p'} - \frac{M_2}{p'}.$$

Since R is equal to the vertical shear in the section, the last member of the preceding equation affords a useful method of obtaining the vertical shear when the moments are known.

In Fig. 102 the force triangle hfi gives the magnitude and direction of a force acting in the diagonal which is in equilibrium with P_3 and S_2, while the superimposed force triangle fgk gives the magnitude and direction of a force acting in the same diagonal which is in equilibrium with P_2 and S_1. The polygon $hfgkih$ must therefore express the relation of equilibrium between P_1, P_2, S_1, S, and S_2, or the polygon *hgkih* that between R, S_1, S, and S_2. Following around the triangle in the direction of the known force R as indicated by the arrows, and transferring these directions to the truss diagram in Fig. 101, S_1 and S are found to be in tension and S_2 in compression. It will be observed that the forces in the polygon *hgki* follow each other in the same order as they are found when passing around the segment of the truss. Fig. 103 shows the same construction when the forces are laid off in the reverse order. As will be illustrated later by an example, it is sometimes preferable to use the one and sometimes the other order.

It is evident on inspection that the most convenient and economical construction of the force polygon in Fig. 102 (or in Fig. 103) would be to draw it directly on a large scale truss diagram. In Fig. 104 one such force polygon is drawn

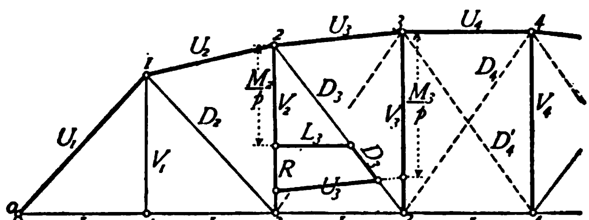


Fig. 104.

in the third panel. The notation shown is well adapted to promote rapid construction and freedom from confusing the stresses. The panels are numbered from left to right and the corresponding numbers are placed at the panel points on their right. The upper chord members are denoted by U, the diagonals by D, and the lower chord members by L, the subscripts being those of the panels containing them. The verticals V necessarily have the subscripts of the panel points. The forces $M_2 \div p$ and $M_3 \div p$, equal respectively to the forces P_1 and P_2 in Fig. 101, are laid off as indicated, and the sides of the force polygon drawn parallel respectively to the truss members whose names are placed by their sides. The panel length p in this case is equal to the horizontal projection p' of the diagonal.

In order to obtain the stress in the vertical V_2, for example, the values of $M_2 \div p$ and $M_3 \div p$ are found for the proper position of the live load (which may not be the same as for D_3) and a force polygon drawn as in Fig. 104. This gives U_3 and D_3 for that position of the load, and on constructing the force polygon for the upper panel point 2 the stress in V_2 is determined. The latter polygon may be drawn directly on the former so as to avoid redrawing the sides U_3 and D_3. To avoid confusion it is omitted in the figure.

This method may also be applied with advantage to determine the stresses in the chords when the moments have been found after placing the live load in its proper position. In this case it will be desirable to place the force polygon on the other side of the diagonal, the values of $M \div p$ being laid off upward from the lower panel points. This will give a polygon like that in Fig. 103.

Prob. 62. Find the maximum live load stress in the counter tie of the fourth panel, and the minimum live load stress in the second vertical, of the truss in Prob. 61.

ART. 51. EXAMPLE—MAXIMUM CHORD STRESSES.

Let the truss in Fig. 104 and on Plate V be that of a single track through railroad bridge, having seven panels each 27 feet long, and depths at the panel points 1, 2, and 3, of 29, 35, and 38 feet respectively. Let the live load consist of Class U of WADDELL'S "Compromise Standard System" (Art. 39). The dead load will be assumed at 1 100 pounds per linear foot per truss, of which 300 pounds is to be applied on the upper chord. This makes the panel loads on the lower chord 21 600 pounds and on the upper chord 8 100 pounds. Since the dead load stresses in all the members of the truss except the verticals are the same whether the dead load is all taken on the lower chord or divided between the chords, and the stresses in the verticals differ by the amount of the upper panel loads (Art. 28), the stresses will preferably be obtained for only lower panel loads of 29 700 pounds, and the maximum and minimum stresses in the verticals afterward corrected by adding to each of them a compression of 8 100 pounds.

In constructing the load line and equilibrium polygon for the live load it was found convenient to use the weights on one rail only, and to adopt scales of 8 feet, 20 thousand pounds, and 2 000 thousand pound-feet per inch respectively. Profile paper, Plate A, was used and required a small splice at the upper right corner in order to extend the curves as far as necessary. In reading the scales the tenths of the small divisions of the profile paper were estimated by eye. When this diagram is used for a double track bridge the stresses obtained are expressed in tons instead of in units of one thousand pounds.

The dead load stress diagram for panel loads of 29.7 thousand pounds is shown in Fig. *b* on Plate V, and the stresses are marked on the diagram. The character of the web stresses is also indicated as referred to the small truss diagram. The computation of the bending moments at the panel points may be arranged as follows, when the panels are all equal: The product of the panel load and panel length is $29.7 \times 27 = 801.9$. The half products of the number of panels in each segment into which the panel points respectively divide the truss are $\frac{1}{2}(1 \times 6) = 3$, $\frac{1}{2}(2 \times 5) = 5$, $\frac{1}{2}(3 \times 4) = 6$; and the bending moments are

$$M_1 = 3 \times 801.9 = 2\ 406, \quad M_2 = 5 \times 801.9 = 4\ 010,$$

$$M_3 = 6 \times 801.9 = 4\ 811 \text{ thousand pound-feet.}$$

Since $p = 27$ feet, the corresponding values of $M \div p$ are:

$$\frac{M_1}{p} = 89.1, \quad \frac{M_2}{p} = 148.5, \quad \frac{M_3}{p} = 178.2 \text{ thousand pounds.}$$

The following table shows the position of the live load obtained by means of the load line. The moments in the third column were read from the moment diagram, and those in the fourth column were copied from the same diagram, while the quantities in the fifth and sixth columns were computed from those in preceding columns. The moments are expressed in units of one thousand pound-feet.

CENTER OF MOMENTS.	WHEEL AT SECTION.	MOMENT AT RIGHT SUPPORT.	MOMENT AT SECTION.	BENDING MOMENT (M).	$\frac{M}{p}$.	REMARKS.
1	4	40 980	480	5 374	199.2	
2	8	39 400	2 622	8 635	319.8	
3	12	39 010	6 331	10 388	384.7	D_4 acts
4	14	32 520	8 291	10 292	381.2	D_4' acts
	15	35 540	10 099	10 210	378.1	D_4 acts

It is necessary to know which diagonal acts in the middle panel for the last three positions in order to determine which moments give the stresses in the chords of that panel. As explained in Art. 47, the vertical shears are found as follows:

$$(39\,010 - 7 \times 1\,470) \div 189 - 166 = -14.1;$$
$$(32\,520 - 7 \times 830) \div 189 - 136 = +5.2;$$
$$(3\,554 - 7 \times 1\,240) \div 189 - 146 = -3.9.$$

These results enable the remarks to be inserted in the last column of the table.

The values of $M \div p$ are now laid off on the truss diagram in Fig. *c* on Plate V, as there indicated, and the force polygons completed as explained in the preceding article. The scales of the original drawing of Fig. *c* were 6 feet and 80 thousand pounds per inch respectively. The values of the stresses are marked on the polygons. The special attention of the student is called to the fact that since U_3 has its center of moments at the lower panel point 3 the side of the polygon *ad* parallel to U_3 must be drawn through *d*, the extremity of $M_3 \div p$ laid off on the vertical ordinate passing through the center of moments. Similarly, as L_2 has its center of moments at the upper panel point 2, the side *bc* must be drawn parallel to L_2 through c', which lies on the vertical through its center of moments. Strict attention to this statement is especially required when the upper panel points are not directly above the lower ones, in which case the panel points should be numbered in regular order from left to right, no matter on which chord they lie. The chords should then have the same subscripts as their centers of moments.

The side *ab* of the polygon *abcd* is not the stress in the diagonal D_3, because the moments at 2 and 3 used in its construction are not simultaneous.

If it be desired to find by this method whether D_4 or D_4' acts when the moment is a maximum at panel point 3, it can be done by finding the simultaneous value of $M_4 \div p$. It is found to be 370.7. If D_4 be assumed to act, the side L_4 will lie below U_4. It is shown as a broken line. By referring again to Fig. 101 the direction around the polygon is toward the right of L_4, upward on D_4, and toward the left on U_4. Upward on D_4 means also toward the right, or away from the section, and therefore tension, which proves the assumption to be correct. Again, since $(M_4 \div p) - (M_3 \div p) = R$, which equals the vertical shear in the section indicated in Fig. 101, the vertical shear in the middle panel is $370.7 - 384.7 = -14.0$, the difference of 0.1 from the value given above being mainly due to the neglect of decimals. Usually the value of the vertical shear is not desired, but simply its sign, in which case it may be known as soon as it is seen whether M_4 is greater or less than M_3 (see also Art. 9).

As the end post receives its maximum stress under the same position of the live load as L_1, its stress may be found in connection with the chords. Indeed, it may be regarded as an upper chord member, the polygon of forces becoming a straight line, as shown on the drawing (Fig. *c*, Plate V).

The following table gives the maximum stresses in the end post and the chords due to the dead and live loads only, expressed in units of one thousand pounds. The minimum stresses equal the dead load stresses.

CHORD MEMBERS.	U_1	U_2	U_3	U_4	$L_1 = L_2$	L_3	L_4
Dead load...	− 121.7	− 117.2	− 127.2	− 126.4	+ 83.0	+ 114.4	+ 126.4
Live load...	− 271.0	− 252.2	− 275.3	− 273.5	+ 185.4	+ 247.0	+ 268.8
Maximum...	− 392.7	− 369.4	− 402.5	− 399.9	+ 268.4	+ 361.4	+ 395.2

Prob. 63. A truss of the same type as that in the example given in this article has nine panels, each 24 feet 9 inches long. The depths at its panel points 1, 2, 3, and 4 are 27′ 0″, 35′ 9″, 41′ 0″, and 42′ 9″ respectively. The dead load is 1 200 pounds per linear foot, of which three-eighths is to be applied at the upper chord. The live load is WADDELL'S Class U. Find the maximum and minimum stresses in the chords and end post due to these loads.

ART. 52. EXAMPLE—MAXIMUM STRESSES IN DIAGONALS.

The first step in finding the live load web stresses is to find the point of division in each panel where a concentrated load will produce no stress in the diagonal (see Art. 48). In practice only that portion of each triangle $B_1B_2B_4$ lying below *cd* in Fig. 98 need be drawn after the vertices on the ordinates at *a* and *b* are marked off. These points are shown in Fig. *d* on Plate V. In case more than one point is shown in any panel the left hand one belongs to the diagonal. The data in the following table are obtained in the same manner as for the chords after the positions are determined. That relat-

Panel.	Wheel at Right End of Panel.	Moment at Right Support.	Moment at Right End of Panel.	Bending Moment (M) at Left End of Panel.	Bending Moment (M) at Right End of Panel.	Left $\frac{M}{p}$.	Right $\frac{M}{p}$.
0—1	4	40 980	480	0	5374	.0	199.2
1—2	3	28 830	230	4117	8004	152.5	296.4
2—3	3	19 840	230	5669	8273	210.0	306.4
3—4	3	12 280	230	5263	6787	194.9	251.4
4—5	3	6 331	230	3618	4292	134.0	159.0
	2	5 501	80	3143	3849	116.4	142.5
5—6	3	2 510	230	1793	1921	66.4	71.1
	2	1 960	80	1400	1600	51.8	59.3

ing to the first panel is inserted here, although it was also included in the table in the preceding article, the end post being treated as a chord member. The moments are expressed in units of one thousand pound-feet. The panels are indicated by the panel points in this table as a guide to the subscripts which properly belong to the corresponding values of M. The moments at the right support for the panel 4–5 are given with greater precision than the rest because it happened that these had been computed and marked on the diagram.

Attention is again called to the fact that the vertical shear in any panel may be found by taking the difference between the corresponding quantities in the last two columns.

In testing for position in panel 5–6 it was noticed that the thread just touched the edge of the step when wheel 3 was placed at panel point 6. When wheel 3 is placed at panel point 5, wheel 1 is a little on the left of the point of division, but the condition of loading is satisfied. If the chords were both horizontal the positions would be 4, 4, 3, 3, 2 and 2 in the successive panels, no panel having two positions of the live load.

The values of $M \div p$ are next laid off on the verticals through the panel points in Fig. *d* of Plate V. The values belonging to each panel are marked inside of the panel to guard against confusion. This danger is not great, however, as it will be noticed that at each vertical the ordinate referring to the panel on the right is considerably less than that for the panel on the left. After completing the force polygons the stresses in the diagonals are scaled off and marked on the diagram. As the portion of the truss on the left of the section through any diagonal is considered, and the lower chord is always in tension, the direction of passing around the polygon is toward the right on L and toward the left on U, and therefore if the direction along D is toward the right it indicates tension. This is seen to be the case for all but one of the polygons shown on the plate.

In the fifth panel two polygons are drawn, the left one for wheel 3 at panel point 5, and the right one for wheel 2 at 5. The latter is placed on the other side of the diagonal to avoid interference. This position is not convenient for diagonals, as will be shown in the next article. In the sixth panel both polygons refer to the position of wheel 2 at panel point 6, the left hand one being drawn in order to show the influence on the construction by changing diagonals. The stress in D_6 when P_2 is at 6 is -21.8.

The maximum and minimum stresses expressed in units of one thousand pounds are given in the following table, the end post being omitted, as its stresses were given in the preceding article.

DIAGONALS.	D_2	D_3	$D_4(=D_4')$	$D_5'(=D_3')$
Dead load	$+46.2$	$+19.6$	0	-20.7
Live load from the right	$+127.5$	$+91.0$	$+69.6$	$+47.1$
Live load from the left	-22.2			
Maximum	$+173.7$	$+110.6$	$+69.6$	$+26.4$
Minimum	$+24.0$	0	0	0

Prob. 64. Find the maximum and minimum stresses in the diagonals in Prob. 63 in Art. 51.

ART. 53. EXAMPLE—MAXIMUM STRESSES IN VERTICALS.

The position for the maximum stress in V_1 is either wheel 4, wheel 12, or wheel 13 at panel point 1 (Art. 46). When wheel 13 is at 1, the same wheels of the second locomotive are placed on the first two panels and in the same position as those of the first locomotive when wheel 4 is at panel point 1, together with one additional wheel; hence it is not necessary to find the stress due to the latter position. The greatest stress occurs when wheel 13 is at 1, and equals

$$[3\,730 - (2 \times 780)] \div 27 = 80.4 \text{ thousand pounds tension.}$$

If it is desired to employ a similar method for V_1 as that described in the preceding article, let the first two panels be regarded as a truss, and the load placed in proper position for maximum moment at panel point 1. The bending moments are then $M_0 = 0$, $M_1 = \frac{1}{2}(3\,730) - 780 = 1\,085$, and $M_2 = 0$. $M_1 \div p = 1\,085 \div 27 = 40.2$. The vertical shear in each panel (disregarding signs) is therefore 40.2, and the floor beam reaction, or the stress in V_1, equals their sum, or $40.2 + 40.2 = 80.4$ thousand pounds.

The respective points of division in the third, fourth, and fifth panels, where a concentrated load produces no stress in the vertical at the left of the panel, are the right hand ones shown in Fig. *d* of Plate V. The position and other necessary data given in the following table are found in exactly the same way as for the diagonals.

PANEL.	WHEEL AT RIGHT END OF PANEL.	MOMENT AT RIGHT SUPPORT.	MOMENT AT RIGHT END OF PANEL.	BENDING MOMENT (M) AT LEFT END OF PANEL.	BENDING MOMENT (M) AT RIGHT END OF PANEL.	LEFT $\frac{M}{p}$.	RIGHT $\frac{M}{p}$.
2—3	2	18 340	80	5 240	7 780	194.1	288.1
3—4	2	11 050	80	4 736	6 234	175.4	230.9
4—5	2	5 501	80	3 143	3 849	116.4	142.5

If the greatest live load stress in V_2 were due to the same position of the load as for D_2, it would only remain to draw (on the diagram in the third panel of the truss in Fig. *d* of Plate V) the line marked U_2 parallel to that member of the truss in order to complete the force polygon for the upper panel point 2. The magnitude and character of the simultaneous stress in V_2 is marked on the diagram. If a force polygon like that one be drawn for the values of $M \div p$ in the first line of the above table, the stress in V_2 is found to be — 61.2 thousand pounds. The construction is omitted on the plate to avoid confusion, as it would partly cover the diagram already drawn. In the same way the greatest live load compression in V_3 is obtained. Its value is — 41.5. As the stress in V_4 (— 26.2) is less than that in V_2, and since V_2 and V_4 are symmetrically located in the truss, the compression to be used for V_4 is the same as for V_2 and will occur when the live load comes on the bridge from the left.

A reference to the lower force polygon in panel 5 of the same figure will now explain why it is not desirable to place all the force polygons for the web members on the right of the diagonals. It is seen that the side of the polygon giving the stress in V_4 lies on the vertical V_5, which is not a convenient arrangement. In the next panel the polygon is placed on the right of D_6 so as to preserve uniformity in laying off the values of $M \div p$ downward, and as no construction is needed for the stress in the vertical adjacent to that panel. The compression in the verticals is usually not required on the right of the middle of the truss.

VERTICALS.	V_1	V_2	V_3
Dead load	+ 29.7	— 4.1	+ 14.0
Live load	+ 80.4	— 61.2	— 41.5
Correction for division of dead panel loads.	— 8.1	— 8.1	— 8.1
Maximum	+ 102.0	— 73.4	— 35.6

The maximum stresses are given in the preceding table, the correction being applied on account of having taken the dead panel loads, as explained in Art. 51. The stresses are expressed in units of one thousand pounds.

Prob. 65. Find the maximum stresses in the verticals in Prob. 63 in Art. 51.

ART. 54. EXAMPLE—MINIMUM STRESSES IN VERTICALS.

In Arts. 30 and 35, as well as in other places, attention has been called to the fact that the stresses in the verticals of a truss with counter-braced panels depend upon the diagonals which are acting simultaneously in the adjacent panels. The influence of the diagonals not only affects the magnitude and character of the stress for any given position of the live load, but also the rate of change in the stress as the live load passes across the bridge. In order to determine what position of the live load will produce the minimum stresses in the verticals of the truss employed in the three preceding articles, let the

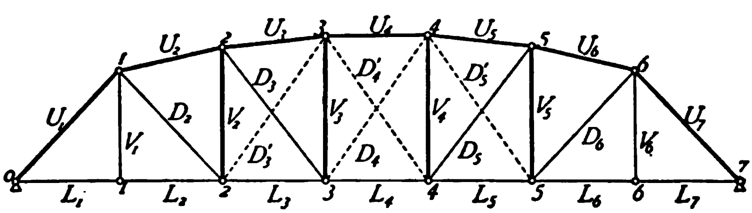


Fig. 105.

complete cycle of changes in the stress in V_3 (Fig. 105) be traced as the locomotives and train pass across the bridge from right to left.

When wheel 1 is at the right support the stress in V_3 is simply that due to the dead load. As the live load advances the combined dead and live load stress in V_3 gradually diminishes at an increasing rate, until a position is reached when the stresses in both diagonals D_4 and D_4' are zero. Mean-while the stress in the vertical has passed through zero from compression into tension. The tension increases at a reduced uniform rate until the stresses in D_6 and D_6' are both zero. If this is not possible, then the tension increases until D_6 becomes a minimum. As the load advances the tension in V_5 at first diminishes and afterwards increases until the stresses in D_6 and D_6' again become zero (if possible). During this interval the stress in V_5 has passed through zero twice, so that it is again tension, but larger in magnitude than before. The rate of change at the beginning and end of the period are also more rapid than in either the preceding or the succeeding one. The tension now increases at a reduced uniform rate until the stresses in both D_6 and D_6' are again zero. As the load advances until it covers the entire bridge, the stress in V_5 diminishes and passes through zero the fourth time and into compression. The rate of decrease was itself a decreasing one being greater at the beginning of this period, and nearly if not quite zero at the end. It will now be very slightly reduced until the head of the train arrives at the left support, when it will remain constant until the rear of the train begins to pass over the bridge. As the train continues to pass off the bridge the compression in V_5 increases at a variable rate until it reaches the maximum value, and then gradually diminishes again to the value of the dead load stress. The absolute maximum compression in V_5 was not reached in this passage of the live load, but will occur when it crosses the bridge from left to right.

During this cycle there were several periods during which the diagonals whose upper extremities are at panel point 5 were not acting, and when the stress in V_5 was therefore to be obtained by drawing the force polygon for that upper panel point. It is evident then that the tension in V_5 is the greatest when the compression in U_5 and U_6 is the largest possible without calling D_5 into action. As the maximum stresses in U_5 and U_6 occur when the entire bridge is covered with the live load, the required position may be obtained from this one by moving the train backwards until the main diagonal in the panel which is adjacent to the vertical, and on the side toward the middle of the bridge, shall just cease to act. For deck bridges this statement would, of course, need modification. In the present example D_5' does not act under any position of the live load, but the statement in the preceding paragraph was so framed as to apply also to V_4 by making the corresponding changes in the subscripts of D and D'.

The required position for the greatest tension in V_5, or its minimum stress, was found by trial to be that when wheel 1 is 3 feet to the left of panel point 4. The moment at the right support is 9860, and those at panel points 4 and 5 are 30 and 1460 thousand pound-feet respectively. The live load bending moments at these points are therefore 5604, and 5583. Adding those due to dead load (Art. 51), $M_4 =$ 10 415, and $M_5 = 9593$. When divided by the panel length of 27 feet, the quotients are 385.7 and 355.3 thousand pounds. When these are laid off on Fig. *c*, Plate V, the resulting force polygon is reduced to two straight lines, indicating that there is no stress in D_5. The corresponding polygon for D_5' is drawn in broken lines. On drawing a parallel to U_5, as shown, and thus completing the force polygon for the upper panel point 5, the stress in V_5 may be measured by scale. The direction of passing around the polygon is evident, since U_4 and U_5 are both known to be in compression. The combined stress is + 30.5 thousand pounds. If M_4 is divided by 38 feet and M_5 by 35 feet, the quotients are both 274.1 thousand pounds, which being the horizontal component of U_4 and U_5 shows also that there is no stress in the diagonals and checks the graphic construction.

After some experience this position can be found with but few trials, and will not require much time if all the operations are performed by graphics. In Fig. 106 let the depths of the truss at panel points 4 and 5 (38 and 35 feet respectively) be laid off on one side of an angle, and some convenient number, as 50, on the other side to the same scale. Join *a* and *b* with *f*. With the same scale which was used in drawing the equilibrium polygon for the wheel loads, lay off the bending moments M_4 and M_5 due to dead load. Now assume a position of the live load, draw the closing line of the equilibrium polygon, and with the dividers transfer the bending moments due to the live load and lay them off above the others. If the position is correct, the lines *ce* and *de*, parallel respectively to *af* and *bf*, will intersect each other on the line *of*. If the head of the locomotive is too far to the left, they will intersect below the line. If *oe* be measured by the scale of moments and divided by 50 feet (assumed as above), the quotient will be the horizontal component of the stresses in U_5 and U_6.

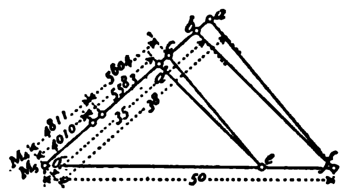


Fig. 106.

If portions of two trains cover certain panels at each end of the bridge, a stress will be caused in V_5 which is a little larger than the value given above, and which can be found as follows: Let the required positions of two trains approaching each other be that illustrated in Fig. 107. The diagonals in

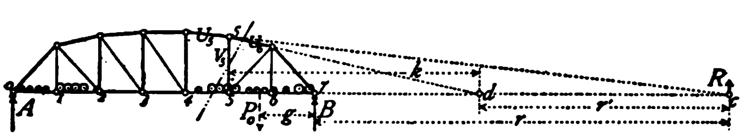


Fig. 107.

the fifth panel are omitted, since there must be no stress in the diagonals of that panel for a maximum tension in V_5, as proved in the preceding portion of this article. Let P_6 be the resultant of the loads transferred to the truss at panel points 5 and 6 by the floor system, together with the dead panel loads at those points, and g its distance from the right support. Let c be the intersection of the chord members U_6 and L_6, and d the intersection of U_6 and L_6. If a section be passed through U_6 and L_6, the stresses in those members hold in equilibrium the forces P_6 and the reaction B, and therefore their resultant. The resultant of the stresses in U_6 and L passes through c; and therefore the resultant R of P and B must be equal and opposite to it, and applied at the same point. The value of R is readily found by taking moments about 7, whence $R = P_6 g \div r$.

If a section be now passed cutting U_6, V_6, and L_6 the stresses in these members hold in equilibrium the same forces P_6 and B as the stresses in U_6 and L_6, since the dead load at the upper panel point 5 is zero in this case. Substituting R for P_6 and B, denoting the stress in V_6 by S, and taking moments about d, there results $- Rr' + Sk = 0$, whence $S = Rr' \div k = P_6 gr' \div kr$. But k, r, and r' are constant; therefore the position of the live load to give the tension in V_6 must be such as to render $P_6 g$ a maximum. This shows that the stress in V_6 is independent of the distribution of the train load on the left, and it may therefore consist of the rear portion of a preceding train.

Referring now to Fig. 108, which shows the truss diagram in position on the load line and moment diagram, the ordinate bf is the moment at the right support of the truss due to the locomotives of the right-hand train. If the chord $4i$ be produced to e, be will represent the moment of the live panel load at 4 about b as a center. The ordinate ef will therefore represent $P_6 g$, less the moment due to the dead panel loads at 5 and 6. As this last moment is constant, ef must be made a maximum. It is also evident that heavy loads should be placed at 5, and usually the head of the locomotive will not pass beyond the panel. The possible positions are therefore quite limited, and on applying the test it is found that when wheel 3 is at panel point 5, $ef = 6330 - (3 \times 230) = 5640$. In this equation 6330 equals bf as read from the diagram, and 230 equals the ordinate $5i$. Similarly, for wheel 4 at 5,

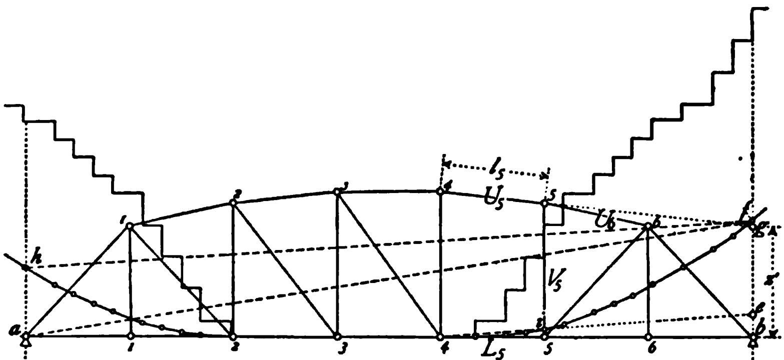


Fig. 108.

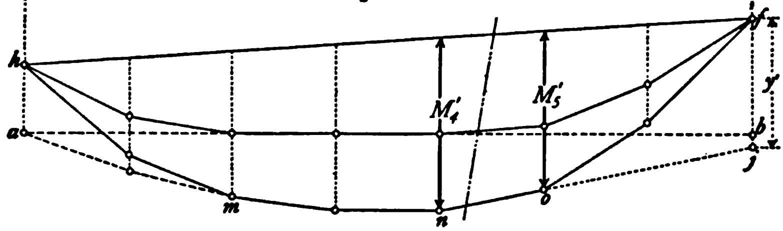


Fig. 109.

$ef = 7260 - 3(480) = 5820$, and for wheel 5 at 5, $ef = 8290 - (3 \times 830) = 5800$. In the required position, therefore, wheel 4 must be placed at panel point 5.

Assuming that the train on the left is also in its right position, the closing line of the equilibrium polygon is hf, while if that train is off the bridge the closing line is af. The equilibrium polygon for the truss is shown in Fig. 109, the ordinates at the panel points being the same as in Fig. 108, and all the sides straight lines. By adding the moments due to dead load below those which are due to live load the polygon becomes *hmnofh*. By Art. 7 the intersection of the sides no and hf produced (Fig. 109) is on the line of action of the resultant R of the forces on the right of the section, and therefore this intersection lies in the same vertical as the intersection of U_5 and L_5. The position of R was shown in Fig. 107. Let the intercept bg at the right support, between the chords U_5 and L_5 produced (Fig. 108), be denoted by z' and that in the same vertical between no and hf produced (Fig. 109) by y', the depths of the truss at 4 and 5 by h_4 and h_5. There follows, $M_4' : h_4 = M_5' : h_5 = y' : z'$, whence

$$\frac{M_5'}{h_5} = \frac{y'}{z'}.$$

If the stress in U_5 be denoted by S', and the angle between U_5 and a horizontal by α, and the length of U_5 by l_5, p being the panel length,

$$S' = \frac{M_5'}{h_5 \cos \alpha} \quad \text{and} \quad \frac{p}{\cos \alpha} = l_5.$$

Substituting,

$$S' = \frac{y'}{z' \cos \alpha} = \frac{py'}{pz' \cos \alpha} = \frac{l_5}{z'} \cdot \frac{y'}{p}.$$

An inspection of Fig. 109 shows that in order to determine y' it is not necessary to know M_4' and M_5', and therefore not necessary to consider either the weight or the position of the train on the left. When that train is not on the bridge the closing line is af, and therefore the corresponding bending moments M_4 and M_5 will also determine y'. Remembering that the moment bf (Fig. 108) for wheel 4 at panel point 5 was found to be 7260, that the moment $5i$ is 480, and that the bending moments at 4 and 5 for dead load are 4811 and 4010 thousand pound-feet respectively,

$$M_4 = \tfrac{4}{7} \times 7260 + 4811 = 8960,$$

and

$$M_5 = \tfrac{5}{7} \times 7260 - 480 + 4010 = 8716.$$

As M_4 and M_5 are respectively 3 and 2 panel lengths from the right support,

$$\frac{y'}{p} = \frac{(3 \times 8716) - (2 \times 8960)}{27} = 304.7 \text{ thousand pounds.}$$

The stress in U_5 can now be found by graphics in the following manner: On Fig. *c* of Plate V draw the line $7g'$ parallel to U_5 intersecting the vertical V_5 at g'; join g' with the upper panel point 4; lay off $y' \div p = 304.7$ on V_5 as indicated, and draw ij parallel to $g4$. The line $j5$ represents the stress in U_5. The force polygon for the upper panel point 5 can next be obtained by drawing jk parallel to U_6. On measuring $5k$ by scale the stress in V_5 is found to be $+$ 31.5 thousand pounds.

This result may be checked as follows: Let a stress diagram be drawn giving the stress in D_5' (Fig. 105) when the reaction at the right support is 1.0. It is found to be 1.017. By the method described in the preceding article, let the stress in D_5' be found for the above values of M_4 and M_5. Its value is $+$ 22.8. To reduce this stress to zero the reaction at *b* (Fig. 108) must be increased $22.8 \div 1.017 = 22.4$ thousand pounds. This requires a moment at the left support *a* of $22.4 \times 189 = 4234$ thousand pound-feet. If this is to be produced by a train approaching from the left, its wheel 1 must be about half a foot on the left of panel point 2, as shown in Fig. 108. If, however, it be produced by the rear end of a preceding train, the train must cover a distance of 65 feet from the left support. The bending moment $M_4' = 8960 + 3/7 \times 4234 = 10775$, and $M_5' = 8716 + 2/7 \times 4234 = 9926$. If these values are respectively divided by the depths $h_4 = 38$ and $h_5 = 35$ feet, each quotient gives the same horizontal component of 283.6 thousand pounds for U_5 and U_6. See Fig. *c*, Plate V.

The greatest stress in D_5' was produced by wheel 3, being placed at 5. The construction for this position is also given on the Plate, the resulting stress in V_6 being $+ 30.8$ thousand pounds.

The stresses in the diagonals in the center panel become zero the second time when the live load covers the entire bridge, and therefore the greatest tension in V_4 or in V_3 occurs when U_4 has its maximum stress. By laying off $M_3 \div p = 178.2$, which is due to dead load, above d, and constructing the triangle fge the stress in V_3 is found to be $+ 44.3$ thousand pounds. If it were attempted to apply the method outlined above, $y' \div p$ would be 609.8, which would give a stress in U_4 greater than that under full load, which is not possible; and if the position of the train approaching from the left, which reduces the stress in D_4 and D_4' to zero, were determined, it would be found to conflict with that of the other train. The maximum tension occurs under full load only for the vertical adjacent to a center panel, or for the middle vertical of a truss with an even number of panels.

The minimum stress in V_1 occurs under dead load only. The accompanying table gives the final minimum stresses after applying the correction on account of dividing the dead panel loads.

VERTICALS.	$V_1 = V_6$	$V_2 = V_5$	$V_3 = V_4$
From diagram	+ 29.7	+ 31.5	+ 44.3
Correction for division of dead panel loads.	− 8.1	− 8.1	− 8.1
Minimum stress	+ 21.6	+ 23.4	+ 36.2

If the number of panels in the truss were 9 or more, the second vertical from the right support would be adjacent to two panels requiring no counter bracing. In such cases the minimum stress in the vertical is obtained in exactly the same way as the maximum, except that the load covers only the smaller segment of the span.

It is apparent that to secure precise results the methods outlined in the example of the four preceding articles, and illustrated on Plate V, must be drawn to a large scale. Results which shall answer all the purposes of design may, however, be readily secured with reasonable care on drawings which are not unwieldy in size.

Prob. 66. Find the minimum stresses in the verticals in Prob. 63 in Art. 51.

ART. 55. STRESSES DUE TO WIND.

The method of finding the stresses in the upper lateral system of a bridge whose trusses have broken upper chords is exactly the same as for horizontal chords after the panel loads due to the pressure of the wind on the trusses are computed.

In order to illustrate the method of obtaining the stresses in the trusses due to the wind, let the same bridge be employed whose dimensions were given in Art. 51, the distance between the centers of trusses being 16 feet. It is usually specified that for bridges of this span the upper lateral system shall be designed for a stationary wind force of 150 pounds per linear foot. For this example the wind load per panel is therefore $150 \times 27 = 4050$ pounds.

Now let the middle three panels of the upper lateral system be considered as separated from the end panels and supported

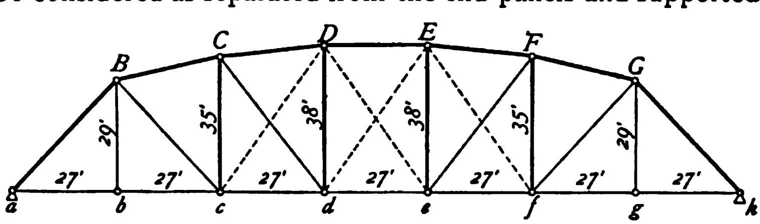


Fig. 110.

at C and F of the windward truss (see Fig. 110), and the corresponding points C' and F' of the leeward truss.

The panel loads at D and E tend to overturn this portion of the upper lateral system, the overturning moment for each half being $4050(38 - 35) = 12\,150$ pound-feet. This must equal the moment of the equal and opposite vertical reactions at C and C' or at F and F', depending upon which half is considered. The lever arm of the couple formed by these reactions is 16 feet, the distance between the centers of trusses. Each reaction equals $12\,150 \div 16 = 760$ pounds, those on the windward side being downward, and on the leeward side upward. The same vertical reactions would be caused by vertical loads of 760 pounds acting downward at D' and E' and upward at D and E. In addition to the vertical reactions referred to above there are also horizontal reactions of 4050 pounds at C and F. Hence the same stresses will be produced in the vertical trusses if the horizontal panel loads of 4050 pounds at D and E are replaced by vertical loads of 760 pounds, acting upward at D and E, and downward at D' and E', together with horizontal wind panel loads of 4050 pounds applied at C and F respectively.

Next consider the entire upper lateral system, with the changes made as just indicated and supported at the points B, G, G', and B'. Horizontal wind loads of $4050 + 4050 = 8100$ pounds are acting at C and F, and these produce an overturning moment in each half of the system of $8100(35 - 29) = 48\,600$ pound-feet. These loads may therefore be replaced by vertical loads of $48\,600 \div 16 = 3040$ pounds, acting upward at B and G and downward at B' and G', together with horizontal wind loads of 8100 pounds at B and G respectively.

The end posts and portal bracing connect the upper lateral system to the supports of the bridge, and may be regarded as a part of it. The horizontal wind loads of $8100 + 4050 = 12\,150$ pounds acting at B and G cause an overturning moment for each half of the bridge of $12\,150 \times 29 = 352\,350$ pound-feet, and may therefore be replaced by vertical loads of $352\ 350 \div 16 = 22\ 020$ pounds, acting upward at B and G and downward at B' and G', together with horizontal wind forces of 12 150 pounds applied at the support of each end of the bridge. It is generally assumed that the feet of the end posts react equally.

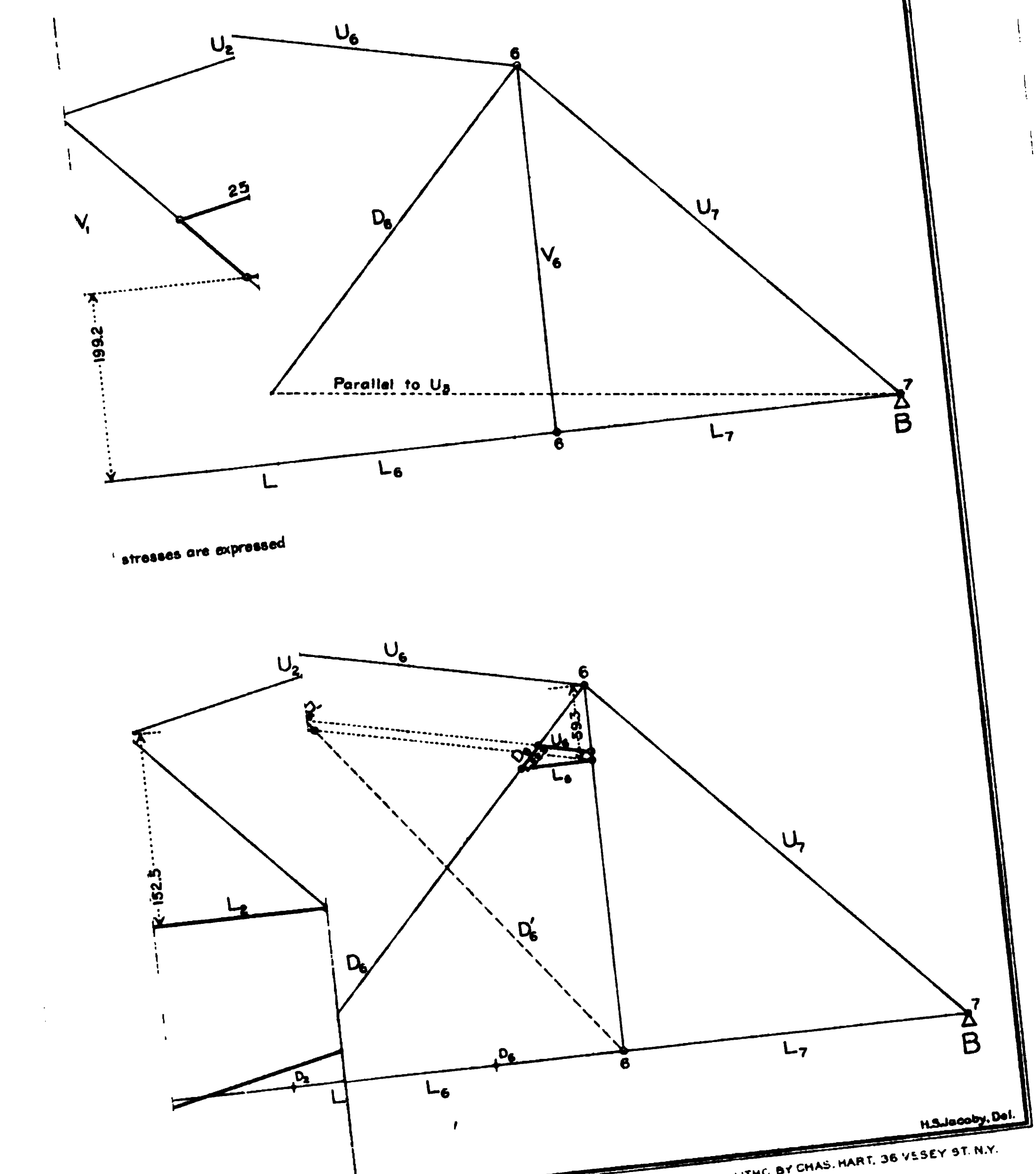
The above analysis shows that the upper lateral system transfers the horizontal wind loads directly to the supports, and that the stresses in the leeward truss due to the overturning moment of the wind pressure which is applied at the panel points of the same system may be obtained by applying vertical loads of 22 020 pounds at B' and G', 3040 pounds at C' and F', and 760 pounds at D' and E' respectively. Since the resulting stresses always act in conjunction with and are less than the dead load stresses, their values in the windward truss are the same as those in the leeward truss with the signs changed. Practically, the wind transfers a part of the dead load from the windward to the leeward truss.

If the upper chord were horizontal the stresses in the truss would be obtained by applying vertical loads only at B' and G' (or at B and G) equal to $3 \times 4050 \times 29 \div 16 = 22\ 020$ pounds. For the sake of simplicity the analysis in this article was made with the implied assumption that the specified wind pressure for the upper lateral system was concentrated at the windward panel points, and that the diagonals take only tension. The wind pressure should, however, be equally divided between the windward and leeward panel points, but the computed vertical panel loads will not be modified by it nor by the fact that the upper lateral system may be designed with stiff diagonals.

If the lower chord be some distance above the level of the bridge supports the wind loads of the lower lateral system will likewise produce an overturning moment. The wind pressure on the train also tends to overturn the bridge about an axis through its leeward supports. In obtaining the transfer of load from the windward to the leeward supports of the bridge the lever arm of the wind pressure equals the difference in elevation between the center of pressure on the train and the axis of rotation. The elevation of the center of pressure is variously specified to be from 7½ to 8½ feet above the base of the rail. The vertical panel loads which may replace the wind pressure on the train in finding the stresses in the trusses must be applied at the panel points of the loaded chord and regarded as a moving load. Each of these panel loads equals the product of the wind pressure on one panel length of the train by the distance of the center of pressure above the plane of the lateral system divided by the distance between the trusses.

The stresses in the trusses, the floor, and lateral systems of a bridge due to the curvature of the track are treated in a paper by WARD BALDWIN in Transactions of the American Society of Civil Engineers, Vol. XXV, page 459, Nov. 1891, entitled "Stresses in Railway Bridges on Curves." The paper contains a practical example in which the stresses are computed.

Prob. 67. Find the stresses in the truss in Fig. 110 due to the wind panel loads as computed in this article. The left end of the truss is toward the north.



H.S.Jacoby, Del.

LITHO BY CHAS. HART, 36 VESEY ST. N.Y.

CHAPTER VI.

MISCELLANEOUS TRUSSES.

ART. 56. THE PEGRAM TRUSS.

The Pegram truss has a curved or broken upper chord, and posts whose angles with the vertical increase from the middle of the truss to its ends. The horizontal projection of the upper chord is about one and one-half panel lengths shorter than the lower chord, but both chords are divided into the same number of panels. The panel points of the upper chord lie upon the arc of a circle. The form, proportion, and relative economy of this type of truss is discussed by the inventor in Engineering News, Vol. XVIII, pages 414 and 432, December 10 and 17, 1887.

Fig. 111 shows the skeleton diagram of a Pegram truss of seven panels. On account of the inclined posts the notation is necessarily modified from that used in Chap. V. The

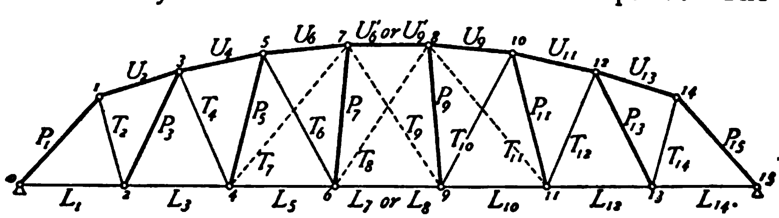


Fig. 111.

panel points are numbered in regular order from left to right. The subscript for any chord member is the number of the panel point which is the center of moments. The subscript for any post or tie is the panel point at the right extremity of the member. The middle panel of the lower chord will be designated by L_7 or L_8, according as its center of moments for a given position of the load is at 7 or 8. In order to distinguish the middle panel of the upper chord from the adjacent members which have their centers of moment at the points 6 or 9, the designating letters are primed.

When the center of moments for any chord member of a truss is not in the same vertical as a floor beam, the method of determining the position of the locomotive wheel loads and the corresponding maximum moment described in Art. 47 does not apply. In Part I, Art. 61, the required criterion for position,

$$P' + \frac{q}{p}Q = \frac{l'}{l}W,$$

was deduced, in which Q is the load in the panel cut by the vertical through the center of moments, P' the load on the left of this panel, W the whole load on the truss, q the horizontal distance from the center of moments to the left end of the panel containing Q, p the panel length, l' the distance of the center of moments from the left support, and l the span of the truss.

In order to satisfy this criterion a wheel has in most cases to be placed at the left end of the panel containing the aggregate load Q, although it will often be satisfied when a wheel is placed at the right end of this panel.

Fig. 112 shows the left end of a Pegram truss in position on the load line and moment diagram of the wheel loads for the maximum moment at the panel point 3. The line *au* produced passes through the point where the vertical at the right support cuts the load line. Wheel 4 is at the floor beam at panel point 2. The ordinate $4r$ represents the load $P' + Q$. If wheel 4 be just to the right of the floor beam the ordinate $2i$ or the equal ordinate mj represents P', while *md* represents $P' + \frac{q}{p}Q$. If, however, wheel 4 be just on the left of the floor beam, the ordinate $2h$ equals P' and mf equals $P' + \frac{q}{p}Q$. The ordinate me equals $\frac{l'}{l}W$ and the position is therefore correct when wheel 4 is at the floor beam and the point e falls between the points d and f where the lines ri and rh respectively cut the vertical through the

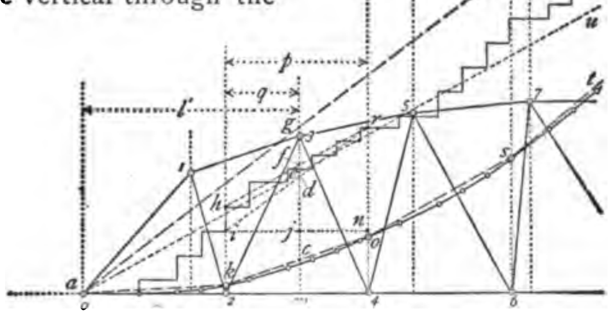


Fig. 112.

center of moments 3. The point r is at the intersection of the load line with the vertical through panel point 4.

With the load in this position the equilibrium polygon for the truss is composed of the straight sides *ak*, *ko*, *os*, *st*, etc., and the closing line *av*. The bending moment is therefore given by the ordinate *cg*. In a similar manner the positions of the live load and the corresponding moments are obtained for all the panel points of the upper chord of the given truss.

The position of the live load and the bending moments for the sections through the panel points of the lower chord are determined in the manner described in Art. 47. For those points the second term of the left-hand member of the above criterion for position becomes zero.

As an example a truss of seven panels will be used whose span is 207 ft. The horizontal projections of the posts P_1,

P_3, P_5, and P_7 are 22.500, 15.595, 9.336, and 3.467 feet respectively, while their vertical components measure 25.000, 32.349, 37.273, and 39.717 feet. The live load consists of WADDELL'S Class U. The dead load is estimated at 1050 pounds per linear foot per truss, 300 pounds being taken on the upper chord. As the average horizontal projection of the upper chords is 23.14 feet, the upper panel loads are $23.14 \times 300 = 6942$, or say 6950 pounds. The panel loads on the lower chord are $750 \times 207 \div 7 = 22\,180$, or say 22 200 pounds.

The following table gives the data for all the live load chord stresses, the moments being expressed in units of a thousand pound-feet, the stresses in thousands of pounds, and the lever arms in feet:

CENTER OF MOMENTS.	WHEEL AT SECTION.	MOMENT AT RIGHT SUPPORT.	MOMENT AT SECTION.	BENDING MOMENT.	LEVER ARM.	STRESS.	CHORD MEMBER.
2	4	47 220	480	6 266	25.96	− 241.4	U_2
4	9	46 570	3 245	10 061	34.53	− 291.3	U_4
6	12	43 070	6 331	12 128	39.14	− 309.9	U_6
					39.717	− 305.4	U_6'
	13	45 090	7 261	12 063			
9	14	35 420	8 291	11 949	39.717		
	15	38 500	10 099	11 901		———	
	16	40 470	11 286	11 840		———	
1	Wheel 4 at point 2			4 790	25.00	+ 191.6	L_1
3	Wheel 4 at point 2			8 270	32.349	+ 255.6	L_3
5	Wheel 13 at point 6			10 710	37.273	+ 287.3	L_5
7	Wheel 12 at point 6			12 090		———	
	Wheel 13 at point 6			12 000		———	
8	Wheel 16 at point 9			11 880	39.717	+ 290 1	L_8

The upper part of this table is arranged like that in Art. 47. For wheel 14 at the section through panel point 9, the shear is positive in the middle panel, while for the two preceding and the two succeeding positions the shear is negative. The moments at 9 for the last two positions cannot be used, since the diagonal in the middle panel which extends to 9 is not then brought into action. The stresses in the fifth and sixth lines of the table are not computed, since the corresponding bending moments are less than 12 128 thousand pound-feet.

The bending moments in the lower part of the table were read directly from the diagram after drawing for each position the closing line of the equilibrium polygon, and the side of the polygon which lay below the center of moments. For the last three positions the shear in the middle panel is negative, and hence only the last moment can be used to obtain the stress in the lower chord of that panel.

Since the ties of this truss do not have equal horizontal projections as in the case treated in Chap. V, it is found to be more convenient to obtain the chord stresses directly by dividing the bending moments by the corresponding lever arms. As the lengths of all the members of the truss must be computed, the lever arms may be obtained by very little additional computation, or they may be measured on a large scale truss diagram with sufficient precision.

The stresses due to the dead load in the upper chord are −105.8, −132.7, −140.8, and −139.4 thousand pounds, and in the lower chord, +85.3, +116.4, +132.1, and 139.4 thousand pounds.

In determining the positions of the live load for the web stresses, the points of division in each panel are found by the method explained in Art. 48, and the results are marked on the truss diagram on Plate VI. It will be observed that the points for the posts are on the right of those for the ties except in the case of T_{11} and P_{11}, where the counter tie is not required. The positions of the locomotive wheel loads and the remaining data required to construct the force polygons for the web stresses are given in the following table. The same units are employed as in the preceding table.

Let the student make a careful comparison between this table and those in Arts. 52 and 53. Attention is also called to the fact that only for one member was more than one position of the load found to satisfy the criterion. The bending

Web Members.	Panel Points at the Extremities.	Wheel at Panel Point.	Bending Moments (M) at Extremities of Web Member.		Length of Horizontal Projection (p).	Left $\frac{M}{p}$.	Right $\frac{M}{p}$.
			At Left End.	At Right End.			
P_1	0—1	4 at 2	0	4 790	22.500	0	212.9
T_2	1—2	4 at 2	4 790	6 270	7.071	677.3	886.8
P_3	2—3	3 at 4	4 800	7 200	15.595	307.8	461.7
T_4	3—4	3 at 4	7 200	9 370	13.976	515.1	670.3
P_5	4—5	2 at 6	6 150	7 090	9.336	658.7	759.3
		3 at 6	6 600	7 580	9.336	706.8	811.9
T_6	5—6	3 at 6	7 580	9 675	20.235	374.6	478.1
P_7	6—7	2 at 9	5 560	5 750	3.467	1608	1659
T_9	7—9	3 at 9	6 340	7 960	26.104	242.9	304.9
T_{11}	8—11	3 at 11	4 040	4 980	30.038	122.3	150.8
T_{12}	11—12	3 at 13	2 010	2 090	13.976	143.8	149.5
P_{13}	12—13	3 at 13	2 090	2 180	15.595	134.0	139.8

moments in the fourth and fifth columns belong to the sections through the panel points given in the second column, and were read directly from the diagram like those in the lower part of the preceding table for chord stresses.

The ordinates whose values are given in the last two columns are laid off on the truss diagram on Plate VI, and the force polygons drawn as explained in the preceding chapter. On account of the great range of these values, and in order that the force polygons may be drawn to as large a scale as possible, different scales are employed. The original truss diagram was drawn to a scale of 6 feet to an inch, and the following scales were used for the force polygons in regular order from left to right: 60, 200, 100, 200, 200, 80, 300, 50, 30, 30, and 40 thousand pounds to an inch. The stress in P_5 for the position of wheel 2 at panel point 6 is —66.2 thousand pounds, the corresponding force polygon not being shown on Plate VI.

The greatest live load tension in P_7 (or in P_9) occurs either when one or both of the adjacent upper chord members are subject to their maximum stresses. In this example their maximum stresses occur simultaneously when wheel 12 is at panel point 6, and while the tie T_8 is acting. The force triangle at panel point 7 therefore gives the corresponding live load stress in P_7 of + 32.0 thousand pounds.

The greatest tension in P_{11} occurs when wheel 5 is at panel point 11. The moment ordinate at the right support is 9475 and at 11 is 830, whence $P_0g = 9475 - (830 \times 3) = 6985$ (see Art. 54). The bending moments in the sections through 8 and 11 due to the dead load are 5526 and 4575 thousand pound-feet respectively. The bending moments due to the dead load plus the above live load on the right end of the truss are

$$M_8 = 5\,526 + \frac{114.819}{207} \cdot 9\,475 = 10\,782;$$

$$M_{11} = 4\,575 + \tfrac{5}{7} \cdot 9\,475 - 830 = 10\,514.$$

The value of y' is found graphically to be 10 030 thousand pound-feet. The horizontal projection p' of U'_{11} is 23.702 feet, and hence $y' \div p' = 423.2$ thousand pounds. On laying this off on the vertical from panel point 10 to i, and drawing ij parallel to $g8$ as explained in Art. 54, $j - 10$ gives a stress of $-$ 333.7 thousand pounds in U'_{11} at the time when P_{11} is subject to its greatest tension. If the force polygon for panel point 10 be next constructed the stress in P_{11} will be found. At this point the stresses in U'_{11}, U_{11}, and P_{11}, together with the dead panel load of 6.95 thousand pounds, are in equilibrium. The polygon is drawn on the truss diagram, and gives a stress of + 31.4 thousand pounds in P_{11}.

The maximum and minimum stresses in the web members due to the dead and live loads may now be obtained in the usual manner:

For the ties,

	T_2	T_4	T_6	T_9	T_{11}
Dead load	+ 56.9	+ 33.7	+ 17.0	0	− 21.0
Live load from the right	+ 141.2	+ 108.0	+ 89.4	+ 74.5	+ 56.4
Live load from the left	0	− 19.3			
Maximum stress	+ 198.1	+ 141.7	+ 106.4	+ 74.5	+ 35.4
Minimum stress	+ 56.9	+ 14.4	0	0	0

For the posts,

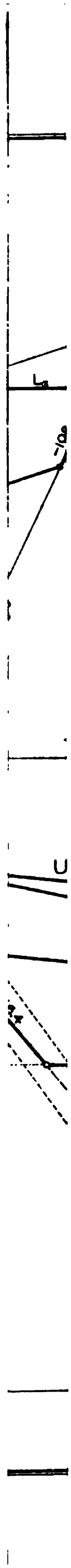
	P_1	P_3	P_5	P_7
Dead load	− 127.2	− 36.2	− 9.1	+ 7.4
Live load from the right	− 286.7	− 108.6	− 67.4	− 39.0
Live load from the left	0	+ 35.5		
Full live load				+ 32.0
Maximum stress	− 413.9	− 144.8	− 76.5	− 31.6
Minimum stress	− 127.2	− 0.7	[+ 31.4]	+ 39.4

It will be observed that if the live load were just a little greater that all the posts except the ones at the end of the truss would be subject to reversal of stress.

Prob. 68. A Pegram truss for a through railroad bridge has five panels, and a span of 150 feet. The horizontal projections of the posts are 20.000, 11.995, and 4.335 feet, while their vertical components are 22.000, 26.505, and 28.740 feet. The live load is WADDELL'S Class U. The dead load is 950 pounds per linear foot per truss, 275 pounds being taken on the upper chord. Find the maximum and minimum stresses in all the members.

ART. 57. THE PENNSYLVANIA TRUSS.

This type of truss is illustrated in the skeleton diagram of Fig. 113. It is derived from the Pratt truss with a curved upper chord by subdividing its panels by means of subverticals



and short diagonals. The vertical broken lines indicate struts which support the upper chord members at their middle points, and the corresponding horizontal lines serve to give lateral support in the plane of the truss to the long vertical posts. These are no real truss members, and are omitted in the diagrams employed in finding the stresses in the truss. In this case the counter diagonal *eG* does not coincide with the short diagonal *FG*, although in a number of trusses which have been erected the panel *eEGg* is counterbraced by

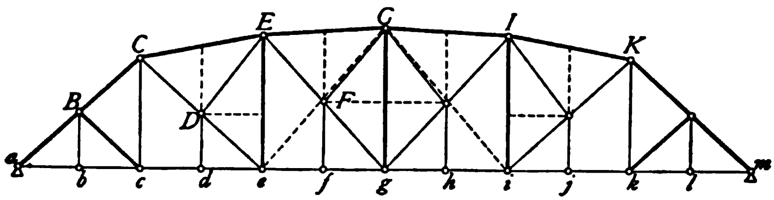


FIG. 113.

connecting the points *e* and *F* with a tie. The detail drawing of such a truss may be found in Engineering News, Vol. XXIII, page 249, March 15, 1890.

The stress in *ef* due to locomotive wheel loads is found in the same manner as for the Pratt truss, the center of moments being at *E*. The stress in *fg* equals that in *ef*, as may be seen from the force polygon for the panel point *f*.

By the method employed in Part I, Art. 61, a criterion for the position of the live load may be obtained which will give the stress in *EG*, and which indicates that the wheel loads from *a* to *e*, plus twice those from *e* to *f*, shall equal $Wl' \div l$, *W* being the whole load on the truss, *l* the span, and *l'* the distance from the left support to the center of moments *g*. To satisfy this criterion a wheel load must usually be placed at *f*, although sometimes it may be satisfied when a wheel is at *e*. In view of the examples given in Chap. V and in the preceding article, the student should have no difficulty in making the graphic construction required by this criterion.

As the section cutting *EG* and only two other members must pass on the left of *f*, the bending moment for *EG* must equal the moment of the left reaction of the truss, minus the moments of the loads transmitted by the floor system to the truss at the panel points *a* to *e* inclusive; or, in other words, the required bending moment exceeds that at the vertical section through *g* by the moment of the panel load at *f*. In the graphic determination, if the line joining the points where the verticals through *e* and *f* meet the moment curve of the live-load diagram, be produced to the vertical at *g*, and the ordinate from this point of intersection to the closing line be read off, the required moment will be obtained.

For the main tie *EF*, the position is found by the method given in Arts. 48 and 49, the auxiliary lines B_1 and B_2 of Fig. 98 being drawn in this case through the points *e* and *f* of Fig. 113. The force polygon is then constructed as in Art. 50 by using the moments at *E* and *g*, the points where the diagonal *EF* meets the upper and lower chords respectively.

The maximum stress in *Ee* occurs also when the head of the locomotive is in the panel *ef*, and hence there will be no simultaneous live-load stress in *DE*. The section through *Ee* must therefore cut *CE* and *ef*, and in finding the point of division in *ef* the chord member *CE* must be produced as in Fig. 98. If in any case the position for *Ee* should be the same as for *EF*, the stress in the former may be obtained from the force polygon already constructed for the latter by completing the polygon for the stresses meeting at the panel point *E*. If not, then a new force polygon for the simultaneous stress in *EF* must be drawn.

The panel load at *f* is suspended from *F* by the subvertical *Ff*, while the members *EG*, *EF*, and *FG* form a secondary truss which serves to transfer this panel load to the panel points *E* and *G*. As the panels are equal one half of the load at *f* is transferred to *E* and *G* respectively. Since the posts

Ee and *Gg* are both vertical, the stress in *Fg* is exactly the same as if the wheel loads in the panels *ef* and *fg* were transferred to the panel points *e* and *g* by a stringer whose span is *eg*. The stress in *Fg* is therefore found by the method given in Chap. V, after considering the members *Ff* and *FG* removed. The greatest stress in the counter tie *eG* (or *Gi*) is obtained in a similar manner to that in *Fg*.

The preceding statement also shows that the maximum stresses in *Ff* and *FG* occur when the floor beam reaction is a maximum. For consolidation locomotives this usually requires the second or third driver to be placed at the floor beam. If the panels are long, the reaction will be greater under the corresponding position of the second locomotive, for one or two of the tender wheels of the first locomotive may then be brought on the panel at the left. The tension in *FG* equals one half of the floor beam reaction multiplied by the secant of the angle which *FG* makes with the vertical.

The stress in the vertical *Cc* depends not only upon the floor beam reaction at *c*, but also upon that at *b*, one-half of the latter being transferred to *c* by the secondary truss *abcB*. By employing the same method as in Art. 46, the following formula may be deduced for the stress in *Cc* due to the locomotive loads:

$$S = \frac{M_d - \frac{3}{2}M_c}{p},$$

in which M_d is the moment of all the loads on the first three panels about *d* as a center, and M_c the moment of the loads on the first two panels about *c* as a center, and *p* the panel length of the three equal panels *ab*, *bc*, and *cd*. The values of M_c and M_d can be read off directly from the live-load moment diagram. The corresponding position of the load requires the wheel loads in the first two panels to be equal to two-thirds of the load on the three panels. It will be observed that this is the same position as that for the maximum moment at *e* of a beam or truss whose span is *ad*. As the live-load diagrams are always constructed with the head of the locomotive toward the left, the maximum stress should also be found in *Kk* and compared with that in *Cc*, the larger value being used for both members. The tension in *Bb* equals the floor-beam reaction at *b*.

If instead of the short tie *FG* a short strut *eF* be inserted, the auxiliary truss will then be *efgF*, which will transfer the panel load at *f* to the points *e* and *g*. The moment of the stress in *fg* about the center *E* will then be the bending moment in the vertical section through *E* plus the moment of the panel load at *f*. The corresponding criterion for position will require the wheel loads from *a* to *f* minus the wheels from *f* to *g* to equal $Wl' \div l$, in which *W*, *l*, and *l'* have the same significance as before, whence *l'* equals the horizontal distance from the left support to *E*, which is the center of moments. The stress in *EG* is the same as if the secondary truss were omitted, and the stringer extended from *e* to *g*. The methods described above for finding the stresses in *EF* and *Fg* will now have to be exchanged.

The construction of the stress diagram for the dead load offers no difficulty in either case, and will therefore not be illustrated.

The form of this truss as shown in Fig. 113 is sometimes modified by reducing the two panels at each end to one, thereby omitting the subverticals at *b* and *l*. This arrangement was used in the Susquehanna River bridge at Havre de Grace, Md. Another modification was adopted in the Merchants' bridge, at St. Louis, whereby the point *B* was raised so as to bring it into the curve of the upper chord. See Engineering News, Vol. XXII, page 578, Dec. 21, 1889.

Prob. 69. The truss in Fig. 113 has a span of 283 feet, the depths at *C*, *E*, and *G* being 42, 47, and 48⅜ feet respectively. Find the stresses in all the members of the truss due

to WADDELL'S Compromise standard, Class U, the bridge having a single track.

ART. 58. THE BALTIMORE TRUSS.

The Baltimore truss is a special case of the Pennsylvania truss when the upper chord is horizontal. The chord stresses for both trusses are found in exactly the same manner. The method used for the web stresses of the Pennsylvania truss also apply to the Baltimore truss, but for most of the web members it is preferable to make the comparison with the methods employed for the Pratt truss.

If the upper chord in Fig. 113 were horizontal the stresses in *EF* and in *Ee* would be the same as if the truss were of the Pratt type with 14 panels, the load on the truss being 14 times the load in the panel *ef*. For the stress in *Fg* the load on the truss must equal 7 times the load in the panel, and the stress would be the same as if the truss were of the Pratt type with only 7 panels. The stresses in *Ff* and *Bb* are equal to those in the suspender of a Pratt truss having the same panel lengths, while those in *Cc* and *FG* are the same as for the Pennsylvania truss.

Prob. 70. A Baltimore truss of a single-track through bridge has the same span, number of panels, and live load as the truss in Prob. 69. its depth being 47 feet 2 inches. The counter-ties *Ge* and *Gi* are, however, omitted, and the members *Eg* and *Ig* counterbraced. Find the live load stresses in all the members.

ART. 59. UNSYMMETRICAL TRUSSES.

Fig. 114 represents the side elevation of the two unsymmetrical Pratt trusses of a through railroad bridge, together with the plans of the upper and lower lateral systems. The floor beams are perpendicular to the center line of the bridge, and they are placed at equal distances apart from each other,

and from the points midway between the bearings of the end stringers. The end posts are inclined so that their horizontal projections are equal to the distance between the floor beams, and this necessitates shortening the end panel of the upper chord at one end of each truss, and lengthening that at the

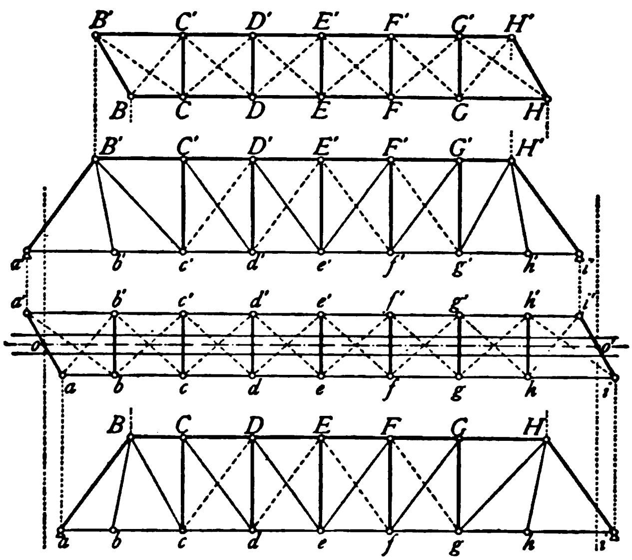


Fig. 114.

other end by an amount equal to one-half the longitudinal component of the skew. The end panels of both chords are therefore equal at the same end of each truss, and the suspenders are inclined. Sometimes the trusses of skew spans have been built whose end posts are not equally inclined, but this is very unusual.

As the two trusses are equal, but with their ends reversed, it is necessary to find the stresses in all the members of one truss for all the loads. Since the load line and equilibrium polygon for the locomotive wheel loads are usually arranged for the load advancing from the right, the live load stresses are found in all the members of the left half of each truss, except the counters whose stresses are found in the right half.

The dead load stresses in the trusses and the stresses in the lateral systems due to that portion of the wind load which is regarded as stationary are as readily determined by the graphic method as for symmetrical trusses, although in the analytic method the numerical work of computation is materially increased.

The stresses in the lateral systems due to the pressure of the wind on the trusses are found by means of a stress diagram like Fig. 67 in Art. 32, but extended to all the members of each system. For the wind panel loads which are treated as a moving load (representing the pressure of the wind on the train), the stresses in the chords may also be obtained by a stress diagram, while that in the lateral struts and ties may be found by means of tables similar to those in Art. 32. The relation between the values in the different lines of the table are, however, not as simple as for trusses whose panels are all equal. For instance, if the nearer truss *ai* is on the windward side, the braces of the lower system brought into action are *ab'bc'cd'de'e . . . hh'i*, and a wind panel load at *g* will cause stresses in members on its left equal to those due to a panel load at *h* multiplied by the ratio of *gi* ÷ *hi*. These products are most conveniently obtained by graphical arithmetic (Art. 14).

Another method, which is preferable in most cases, is to draw a diagram giving the stresses in all the members for a reaction of 1.0 at the left support *a*, and then to multiply the stress in each brace by the corresponding reaction produced by the panel loads which make its stress a maximum. These reactions are most quickly determined by means of an equilibrium polygon whose closing line will shift as one panel load after another is taken away from the full load for which it is at first constructed.

The position of the wheel loads is obtained in the same way as for symmetrical trusses, it being assumed, however, that the loads are distributed along the center line oo' of the track as shown in Fig. 114, and that the trusses have their supports at o and o'. Although this method is approximate, it generally gives the correct position. In case a very slight shifting of the loads would dissatisfy the criterion, it may be well also to find the stress when the next wheel is placed at the corresponding panel point and compare the results.

In the bridge represented in Fig. 114 let the span be 146 feet, $bc = cd = de = ef = fg = gh =$ 18′ 3″, $ab = BC =$ 13′ 7½″, $hi = GH =$ 22′ 10½″, the width 16 feet between centres of chords, and the depth 24 feet. For the greatest stress in the chord members ab and bc due to WADDELL'S Class U (Art. 39) the required position is that with wheel 3 at panel point b. Fig. 115 shows the left portion of the truss diagram placed in this position on the load line and moment diagram of the live load. The intersection of the line ov with the

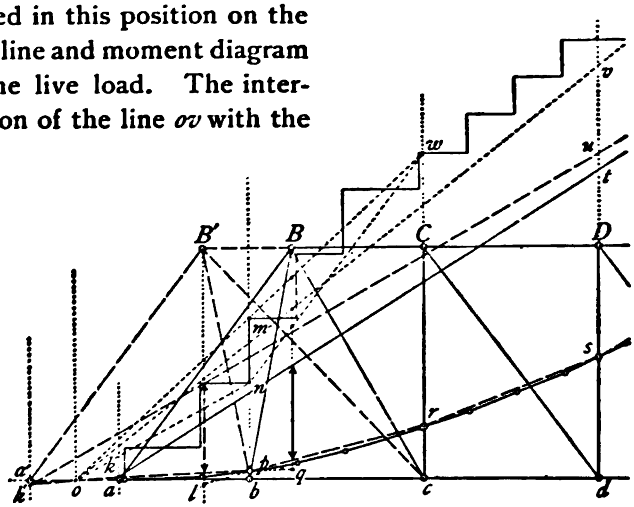


Fig. 115.

vertical through the center of moments B is seen to lie between the points where mw and nw cross the same vertical.

As the floor system distributes the load to the panel points of the truss the lower sides of the equilibrium polygon for the truss under this position of the load are the right lines, or chords, *op*, *pr*, *rs*, etc., the points *o*, *p*, *r*, and *s* being on the live load polygon. The point *o* is the regular panel length of 18′ 3″ on the left of *b*. The closing line of the polygon must end in a vertical through the left support of the truss at *a*, and therefore at the intersection *k* of the vertical *ka* with the line *op*. In a similar manner the right end of the closing line (not shown in Fig. 115) is located at the intersection of the chord or side of the equilibrium polygon whose extremities lie on the verticals through *h* and *o′* (see Fig. 114) with the vertical through *i*. The bending moment in the section through *B* is measured by the full line ordinate with arrows at its extremities (Fig. 115). This moment divided by the depth of the truss gives the stress in *bc*. As the stresses in *ab* and *aB*, however, hold in equilibrium only the reaction at *a*, the moment of the stress in *ab* about *B* is equal to the moment of the reaction, and hence is measured by the ordinate when produced to *q*, its intersection with the side *op* (or *kp*) produced.

The stress in the end post *aB* is preferably obtained by dividing the stress in *ab* by the sine of the angle which *aB* makes with the vertical. For the remaining web members except the suspenders, the only modification required of the method described in Art. 45 for trusses with equal panels is that the moment at the right support must be read to the point corresponding to that described in the preceding paragraph as the right end of the closing line of the equilibrium polygon for the truss.

The floor-beam reaction is the same as if the panel *ab* of the truss were equal to *bc*, for the deduction of the formula in Art. 46 indicates that the panel lengths introduced are really the spans of the corresponding stringers, and while usually they are equal to the panel lengths of the truss, this is not always the case. The tension in *Bb* equals the product of the floor-beam reaction by the secant of the angle which *Bb* makes with the vertical.

Fig. 115 also shows the left-hand portion of the diagram of truss $a'B'H'i'$ superimposed upon the other. The closing line is $k'u$, k' being at the intersection of *po* produced with the vertical $a'k'$. The moment of the chord stress in $a'b'$ (b' coincides with *b*) is measured by the ordinate below B' and is indicated by arrows at its extremities. By producing this ordinate to *l*, on the chord *rp* produced, it gives the moment of the stress in $b'c'$.

It will be observed that the same position of the live load was used for the end chord members of both trusses. If the point *o* had been moved to a' the criterion would not have been satisfied by placing wheel 3 at *b*, but only by putting wheel 4 at *b*. The moment for the latter position, however, is less than that for the former.

Prob. 71. Find the maximum and minimum stresses in the above example due to the given live load and a dead load of 900 pounds per linear foot, one-fourth to be taken on the upper chord.

ART. 60. DOUBLE AND QUADRUPLE SYSTEMS.

For trusses having more than one system of webbing it is assumed that each system is affected only by the loads which it carries.

In the double system Warren truss in Fig. 116, the loads

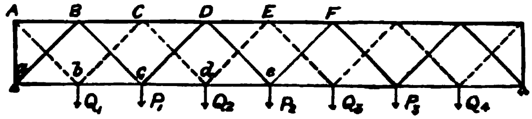


Fig. 116.

P_1, P_2, and P_3 are carried by the full line diagonals, and the loads, Q_1, Q_2, Q_3, and Q_4 by the diagonals drawn in broken lines. The truss is therefore regarded as composed of two separate trusses having common chords. The stresses in each system may then be determined and the results combined. In this case, however, either the dead load stresses in all the members or the live load stresses in the chords may be found by means of one diagram. The reaction at the left support is in equilibrium with the stresses in *Aa*, *Ba*, and *ab*, but the compression in *Aa* is known, since it equals the reaction due only to the loads *Q*, thus leaving but two unknown stresses. The stress diagrams may therefore be readily constructed. The maximum live load stresses in the diagonals are obtained by considering each system separately.

For the Whipple truss in Fig. 117 (which is a double intersection truss of the Pratt type) both dead and live load

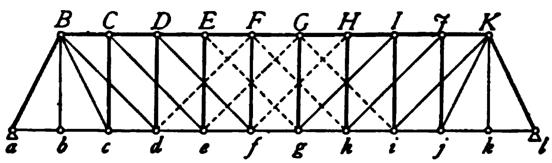


Fig 117.

stresses must be found for each system, the division into systems being somewhat different, however, for the required stresses in the chords and web members. For the chord stresses the division may be made into the two symmetrical systems shown in Fig. 118, provided the live load is uniform

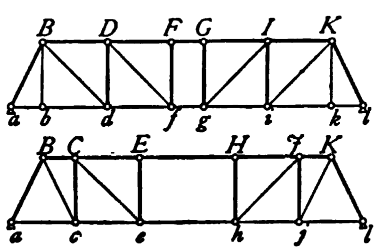


Fig. 118.

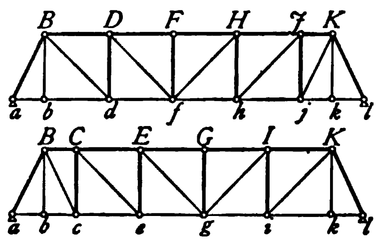


Fig. 119.

throughout. If the live load is not uniform, or if it consists of excess loads combined with a uniform train load, the division may be made similar to that shown in Fig. 119, care being taken to insert those diagonals near the middle, which are in tension under the combined dead and live loads. If the stresses are obtained only for the left half of the truss, the excess loads may require an additional diagonal to slope downward toward the left in the lower diagram of Fig. 119. It is clear that only those dead and live load stresses in any chord member may be added together, which were obtained under the same conditions; that is, with the same diagonal of the given panel acting in each case.

When the division is made into systems, or component trusses, which are unsymmetrical, there is some ambiguity in the stresses due to the fact that the suspenders are attached to the panel points *B* and *K* which are common to both systems. As reasonable an assumption as any is to regard the panel loads at *b* and *k* to be equally divided between the two systems. The same stresses will, however, be obtained if both suspenders be considered as a part of only one and the same system, and this arrangement is also more convenient in finding the stresses.

If these two methods of division be compared for a uniform load throughout, the greatest difference in chord stresses is found to be not quite four per cent, most of them being much less. The difference may be reduced one-half by considering the suspender *Bb* as belonging only to that component truss which contains the adjacent diagonal *Bc*, and the suspender *Kk* as being a part of the system containing the diagonal *Kj*. This arrangement reduces the shear at the middle to the minimum value possible in each case, and requires the construction of only one stress diagram for the chord members, since one system equals the other with its ends reversed.

For the stresses in the web members the systems are divided as in Fig. 120, all the ties sloping one way except those near the right end, where it is certain that no counters are needed.

Only the loads supported by one of the systems are considered in finding the stresses due to both dead and live loads in any web member of that system. The method employed is that

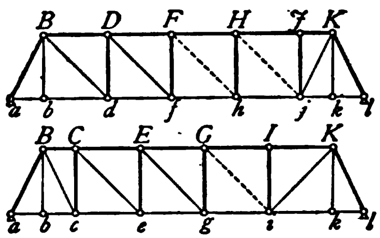


Fig. 120.

of Art. 30, the labor of tabulation being materially lessened by noticing the general statements made in that article in regard to the maximum and minimum stresses in web members. On account of the ambiguity in stresses the panel loads at *b* and *k* may be placed on either system, so as to produce the maximum and minimum stresses in any given web member.

When one excess load is used in connection with uniform panel live loads, it must be placed on that system which gives the greatest chord stresses; and for two excess loads the first may be on one system and the second on the other, depending upon their distance apart and the panel length. For maximum stresses in the webbing the excess loads are always placed at the head of the train.

If concentrated wheel loads are to be employed it will be best to always place the first driver at the panel point. Each system is regarded as acting independently, and as being strained only by the loads transferred to it by the stringers and floor beams. As part of the weight of the pilot is carried by the stringer to the other system, that part is disregarded in obtaining the stresses in the bracing. For the chord stresses the locomotives are so placed as to produce the greatest moment at the middle of the truss, and the weights transferred to each system are used only in determining the chord stresses for that system.

A quadruple Warren truss or lattice girder is treated in

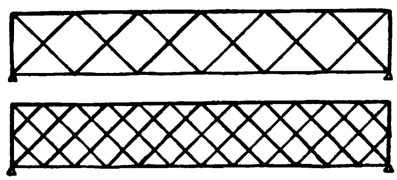


Fig. 121.

a similar manner to the double system Warren, and also requires but one diagram for dead load.

Prob. 72. A double system deck Warren truss of 100 feet span has 10 panels, and is 10 feet deep. The dead load per linear foot per truss is 560 pounds, and the train load 1800 pounds, which is preceded by two heavy locomotive panel loads of 65 000 pounds each. Find the maximum and minimum stresses in all the members.

ART. 61. THE GREINER TRUSS.

The truss shown in Fig. 122 was designed in 1894 by J. E. GREINER, Engineer of Bridges of the Baltimore and Ohio Railroad, for overhead highway purposes so as to be adapted

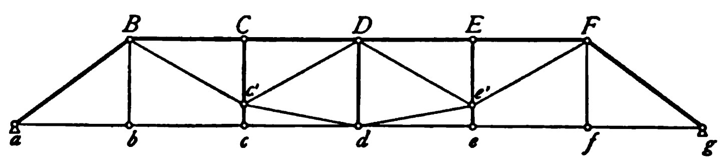


Fig. 122.

to the use of material taken out of old bridges combined with rails worn out in track service. It consists essentially of a bowstring truss with a horizontal upper chord combined with a Pratt truss whose diagonal ties are omitted, so that the supports of the former are at the upper ends of the end posts of the latter.

The stresses in the web members above the bowstring

Bc'de'F are first determined by regarding them as members of the bowstring truss only, the supports being at *B* and *F*. The stresses in the end posts are the same as if the truss were of the usual Pratt type not in combination with the bowstring, while the stresses in the verticals *Bb*, *c'c*, *e'e*, and *Ff* are the corresponding floor-beam reactions.

The stress in any member of the upper chord *BF* is the sum of its stresses when considered successively as a member of the bowstring and Pratt systems, the load in the second case being the panel loads at *b* and *f*, together with loads at *B* and *F* equal to the reactions in the first case. This stress is a maximum when the load covers the entire truss and its position satisfies the same criterion, which would be used if the truss were of the pure Pratt type. The corresponding stresses in the bowstring *Bc'de'F*, and in the lower chord are easily found, as each of these chords belongs only to one of the component trusses.

Since the diagonal bracing of the Pratt truss is replaced by the bowstring truss, it is necessary not only to determine the stresses in each system separately, but also to consider the effect on the latter when the load on the former is not symmetrically disposed. As the panel points *B*, *d*, and *F* are common to both systems, the Pratt truss may first be regarded as having counterbraced long diagonals *Bd* and *dF*, and after the stresses in *Bd* and *dF* are found for the unequal loads transferred to *B* and *F*, or the panel loads at *b* and *f*, or both of these conditions combined, the stress in each of these members may be replaced by two equal and opposite external forces applied at its extremities, as was done in Art. 33, Fig. 69, in replacing the initial tension in the counters. The stresses in the bowstring truss due to these forces must then be combined with those obtained by treating each system independently. Only those stresses must be combined which occur simultaneously under the same conditions.

A more expeditious method is to draw three stress diagrams for the truss for a panel load at *b*, *c*, and *d* respectively. The stresses are then tabulated and combined according to the method given in Art. 29. Each diagonal is designed to resist whatever kind of stress it receives.

If the dead and live loads are uniform and the bowstring is parabolic, the maximum stresses in the two horizontal chords are uniform throughout. This fact indicates why this type of truss is adapted to the use of track rails in these chords. The details of a truss of this type, in which, however, the parabola is in compression, were published in the Railroad Gazette, Vol. XXVII, page 602, Sept. 13, 1895.

Prob. 73. A highway bridge with a clear roadway of 15 feet has a span of 57 feet. The truss is of the same form and number of panels as Fig. 122, and its depth is 7 feet. Find the stresses due to a live load of 80 pounds per square foot.

ART. 62. HORIZONTAL SHEAR IN A BEAM.

Let it be required to construct a diagram showing the distribution of the horizontal shear in a beam. This may be conveniently illustrated by an example, such as occurs in the design of a deepened beam. A deepened beam consists of two timbers of rectangular cross-section placed above each other and united by keys or brace blocks so as to make the timbers act like a single stick. By this means the combined strength of the timbers is double that secured when they act separately.

Let the loads to be supported exclusive of the weight of the beam consist of three concentrated loads of 4000, 8000, and 6000 pounds respectively, and a uniformly distributed load of 2000 pounds per linear foot, extending over a portion of the span as indicated in Fig. 123. A single bending moment diagram for both concentrated and uniform loads

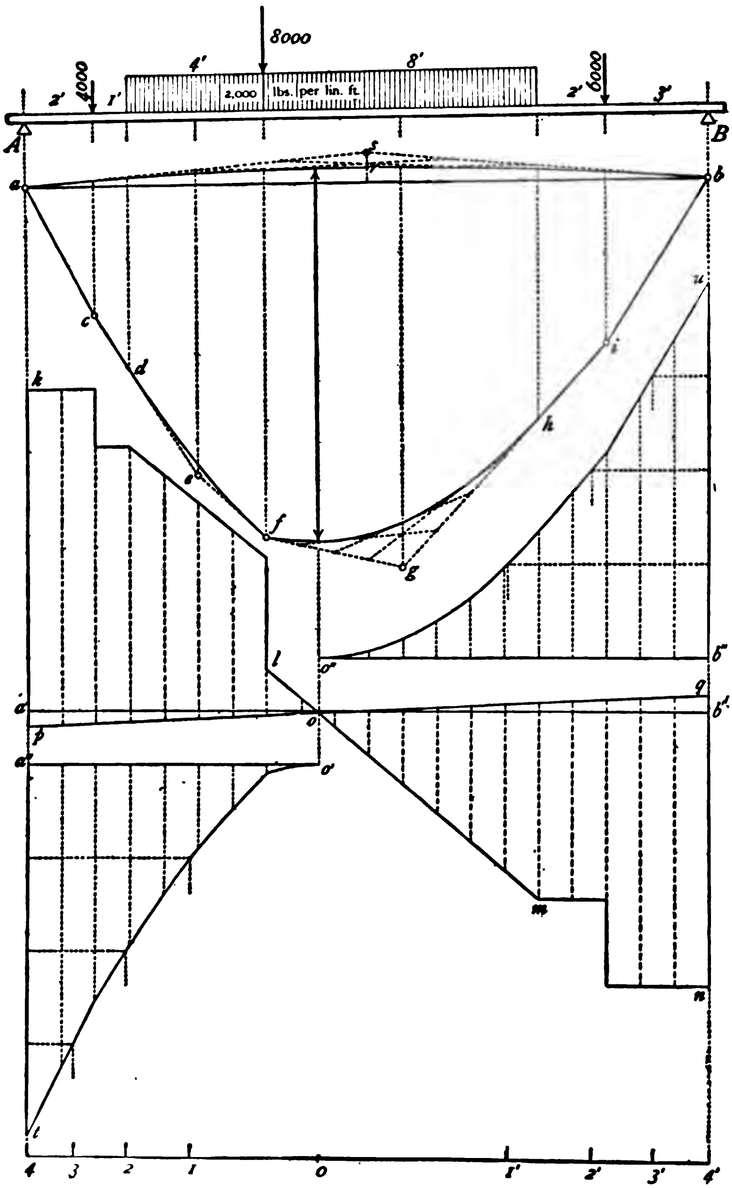

8000
4000
6000
2'
1'
4'
8'
2'
3'
2,000 lbs. per lin. ft.
A
B
a
b
c
d
e
f
g
h
i
k
l
m
n
o
o'
o"
p
q
t
u
a'
b'
b"
4
3
2
1
0
1'
2'
3'
4'

Fig. 123.

may be constructed by treating separately the portions of the uniform load which lie between the concentrated loads. The loads are taken in succession from left to right and laid off on the load line (not shown). The left reaction *A* is found by computation to be 22 900 pounds, and by laying this off on the load line the closing line of the equilibrium polygon may be made horizontal, if so desired, by taking the pole directly opposite the point of division of the reactions (see Arts. 9 and 11). The equilibrium polygon *acefgiba* is first drawn by regarding the portions of the uniform load as concentrated at their centres of gravity. The final form is then obtained from this by constructing a parabola (Art. 10), tangent to the right lines *ce* and *ef* at the points *d* and *f* respectively, these points being directly below the extremities of the 4-foot portion, and a second parabola tangent to *fg* and *gi* at the points *f* and *h*.

The vertical shear diagram *a'k . . . lm . . nb'* is drawn next. The shear passes through zero, where *lm* crosses *a'b'*, and on measuring the moment ordinate directly above this it is found to be 126 400 pound-feet. If *b* be the breadth and *d* the depth of the rectangular section of the beam, and 875 pounds per square inch the working unit stress in the outer fibers,

$$bd^2 = (126\,400 \times 12 \times 6) \div 875 = 10\,400 \text{ inches}^3.$$

If *b* be assumed as about $\frac{1}{2}d$, the beam will require two timbers 14 × 14 inches in section.

The weight of these timbers for a length equal to the span, at 3 pounds per foot board measure, equals 1960, or say 2000 pounds. The bending moment at the middle of the beam due to this weight is 5000 pound-feet. The corresponding moment diagram is drawn as explained in Art. 10 by making the parabola *arb* tangent to *as* and *sb*, the ordinate at *s* being 2 × 5000 = 10 000 pound-feet. The shear diagram *a'pqb'* for the weight of the beam is added to the other by laying off the reactions in the opposite direction from the axis $a'b'$. The shear due to all the loads passes through zero at o where lm crosses pq, which is a little to the right of the point found before, and is 8.52 feet from the left support. The maximum bending moment directly above this point is now measured, and its value of 131 300 pound-feet obtained. As the resisting moment of the beam slightly exceeds this amount no correction is necessary.

If S_h is the unit horizontal shear, and V the vertical shear in any section of the beam, b and d the breadth and depth of the rectangular cross-section, the following relation is given by mechanics (Mechanics of Materials, Art. 78).

$$S_h = \frac{3}{2} \cdot \frac{V}{bd}.$$

In order to obtain the horizontal shear for a distance dx along the beam, S_h must be multiplied by bdx, giving

$$S_h bdx = \frac{3}{2d} . Vdx.$$

If the total horizontal shear be required between any two sections of the beam, it is necessary to integrate this expression between the given limits of x. V is a function of x, and the integral of Vdx is the area of the vertical shear diagram between the given sections.

The total horizontal shear between a' (the left support) and o is $\frac{3}{2d} . M_{max.}$, since $\int Vdx = \int dM = M$. Substituting the values found above, the total horizontal shear for either the portion $a'o$, or ob' equals $(3 \times 131\,300 \times 12) \div (2 \times 28) =$ 84 420 pounds. If four keys or brace blocks of equal strength are to be employed to resist this shear, each block must be designed to take a pressure of 21 105 pounds. In order to determine their location, it is necessary to divide the vertical shear diagram on each side of the zero shear into four equal parts. This is most readily done by dividing it into narrow strips, say a foot wide by scale, finding the area of each one, and, beginning at the point *o*, adding each area to the sum of the preceding ones. The total area should equal the maximum moment. These areas are laid off as ordinates on the axis $a''o'$ in such a way that the length of the ordinate at any section of the beam represents the area of the vertical shear diagram from that section to the point of zero shear. By dividing the last ordinate $a''t$ into four equal parts and drawing parallels to $a''o'$ through these points as indicated by broken lines, their intersection with the curve passing through the extremities of the ordinates gives the positions required.

The corresponding diagram for the right-hand portion is drawn above the axis $o''b''$. All the positions of the brace blocks are marked on the bottom line of Fig. 123. Numbers 1, 2, and 3 are respectively 4′ 9″, 2′ 11″, and 1′ 4½″ from 4, which is at the left support; while numbers 1′, 2′, and 3′ are 5′ 10½″, 3′ 5″, and 1′ 7½″ from 4′ at the right support. It will be observed that in the middle portion of the beam no brace blocks are required in this case for almost one-half of the span.

When moving loads are substituted for stationary loads the length of the middle space is materially reduced, for in that case the maximum horizontal shear in any section does not occur when the load covers the entire beam except for the sections at the supports. If the cross-section of the beam is not uniform, it is necessary to construct the diagram so that the ordinates shall represent the corresponding sums of the horizontal shears directly. By using the general form of the equation the distribution of the horizontal shear in a beam of any cross-section may be similarly shown by a diagram.

Prob. 74. Two deepened beams having an effective span of 22 feet carry a single track railway across a culvert. The weight of the track is to be assumed at 400 pounds per linear foot, and the live load at 3600 pounds per linear foot. The beams are to be of timber weighing 45 pounds per cubic foot, and with an allowable unit stress in the outer fibers of 1500 pounds per square inch. Find the positions of the brace blocks or keys, provided eight are used in each beam.

ART. 63. ROOF TRUSS WITH COUNTERBRACES.

Fig. 124 shows the skeleton diagram of a roof truss with straight upper chords and a curved lower chord. The upper chord on each side is divided into five equal panels, and at the panel point marked 2 the strut is normal to the upper chord. The end panels of the lower chord are parallel to the adjacent upper chord members and the panel points 1, 2, . . . 5 . . . 2′, 1′ lie on the arc of a circle. With the exception of the end ones, the lower chord members are also equal. The ventilator covers a panel on each side of the peak.

The dead panel loads 1 to 7 on the upper chord are 2500, 3350, 3350, 3350, 2750, 2150, and 3600 pounds, and the corresponding loads 0 to 5 on the lower chord are 1400, 600, 600, 600, 600, and 600 pounds, somewhat less than half the weight of the truss being regarded as concentrated at the panel points of the lower chord. The snow panel loads are 2700 pounds, excepting those at 1, 5, and 6 which are 1350 pounds. The horizontal wind pressure against the vertical side 0—1 is 5600 pounds, and against 5—6 is 4800 pounds. The normal wind panel load is 6300 pounds, distributed to the panel points in the usual manner.

The stress diagrams are drawn at first for the truss with its counterbraces omitted as described in Art. 17, the additions required on account of the counters (in this example, counterties) being made afterwards. Only one of the stress diagrams is shown, that due to wind on the fixed side of the truss being given in Fig. 125. As the direction of the reaction at the fixed end is not known, the equilibrium polygon used to determine the reactions must have its left-hand vertex at that support and its right-hand vertex on the vertical line of action of the reaction of the right support. To reduce the number of sides of the equilibrium polygon, and thereby to secure greater accuracy, it is desirable to concentrate the wind pressures on the two vertical and the two inclined surfaces at their respective centers. Should this arrangement cause a vertex of the equilibrium polygon to fall beyond the limits of the drawing even with the most favorable position of the

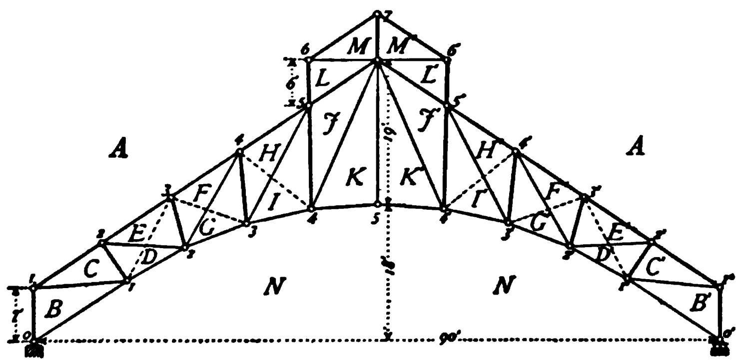


Fig. 124.

pole, it may be remedied by replacing the horizontal and normal pressures on the ventilator by their resultant. To avoid ambiguity in stress, the member AM' is attached to the panel point 7 so as not to receive or transmit any direct stress due to the wind.

To determine the stresses in the members adjacent to the second panel of the truss when the counter-tie DE acts, imagine the main diagonal removed and complete the corresponding force polygon for the panel points at its extremities, the additional lines required being made with short dashes instead of full lines. In a similar manner the additions are made due to the counters FG, HI, and those in the other half of the truss. It will be observed, for instance, that the polygon $hhii$ in the stress diagram has all of its sides respectively parallel to the members of the panel containing the diagonal ties *HI*, while the two points marked *i* are on the same parallel to *NI*, and the two points *h* are on the same parallel to *AH*.

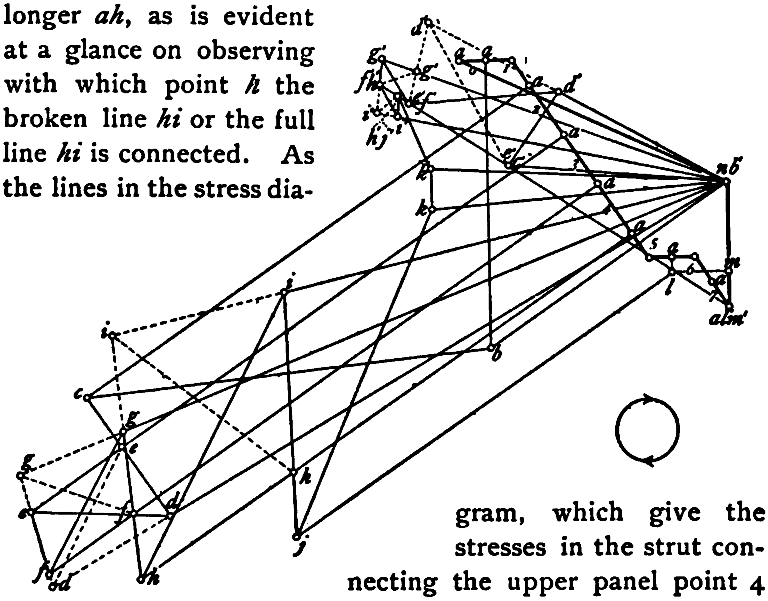


Fig. 125.

The stress in *AH* when the counter acts is represented by the shorter *ah*, while its stress when the main tie acts is the longer *ah*, as is evident at a glance on observing with which point *h* the broken line *hi* or the full line *hi* is connected. As the lines in the stress diagram, which give the stresses in the strut connecting the upper panel point 4 with the lower panel point 3, have the same small or lower-case letters at their extremities as the capitals which designate the spaces adjacent to the strut, an inspection of the truss diagram will indicate which letters are to be used in any given case. When the counter acts on the left and the main tie on the right the strut is designated as *FH* and the corresponding stress is *fh*. If the counters act on both sides the strut is *FI* and its stress *fi*. The character of the lines representing the stresses in the main and counter ties afford a check against errors, for the stress diagram above shows that *gi* is the stress in the strut when the acting adjacent ties are the main tie *FG*, and the counter tie *HI*. As an additional example, the stress in the next strut to the left is *eg* when the acting adjacent ties are the main *DE* and the counter *FG*.

Before measuring the stresses in the chords and struts the stresses in the main and counter ties are tabulated so as to determine how many counters are actually required. The following tables give all the stresses for the ties in three panels and the maximum and minimum stresses in the remaining ties, the stresses being expressed in units of one thousand pounds.

TIES.	*DE*		*FG*		*HI*	
	MAIN.	COUNTER.	MAIN.	COUNTER	MAIN.	COUNTER.
Dead load	+ 17.8	− 19.8	+ 1.3	− 1.0	+ 14.5	− 10.3
Snow load	+ 10.8	− 12.0	+ 1.2	− 0.9	+ 9.4	− 6.7
Wind on fixed side	+ 14.8	− 16.4	+ 16.8	− 12.6	+ 33.7	− 23.9
Wind on free side	− 2.4	+ 2.7	− 8.1	+ 6.1	− 12.7	+ 9.0
Maximum	+ 43.4	0	+ 19.3	+ 5.1	+ 57.6	0
Minimum	+ 15.4	0	0	0	+ 1.8	0

TIES.	*BC*	*B'C'*	*D'E'*	*F'G'* (m)	*F'G'* (c)	*H'I'*	*J'K'*	*JK*
Maximum	+ 98.1	+ 78.4	+ 44.3	+ 16.4	+ 3.1	+ 40.8	+ 47.1	+ 68.3
Minimum	+ 24.4	+ 34.1	+ 14.3	0	0	+ 14.5	+ 18.9	+ 5.9

These tables show that only one panel in each half of the truss needs counterbracing.

The following tables give all the stresses in four chord

CHORD MEMBERS.	*NB*	*ND*	*NG*		*AF*	
			MAIN.	COUNTER.	MAIN.	COUNTER.
Dead load	0	+ 30.5	+ 46.0	+ 46.9	− 52.4	− 51.5
Snow load	0	+ 18.9	+ 27.8	+ 28.7	− 31.9	− 31.2
Wind on fixed side	+ 30.1	+ 67.8	+ 68.3	+ 80.2	− 70.4	− 59.4
Wind on free side	− 33.5	− 42.2	− 37.7	− 43.5	+ 13.6	+ 8.3
Maximum	− 33.5	+117.2	+142.1	——	−154.7	——
Minimum	+ 30.1	− 11.7	——	+ 3.4	——	− 43.2

CHORD MEMBERS.	AC	AC′	AE	AE′	AH	AH′	JL	J′L′
Maximum..........	−122.7	− 93.8	−154.3	−125.7	−144.4	−125.4	−116.2	−109.2
Minimum...........	− 29.2	− 40.8	− 25.4	− 53.3	− 42.9	− 51.2	− 41.7	− 42.7

CHORD MEMBERS.	AF′	NB′	ND′	NG′	NI	NI′	NK = NK′
Maximum	−126.0	− 3.3	+ 69.4	+110.0	+106.3	+ 93.7	+ 75.1
Minimum............	− 52.4	0	+ 30.5	+ 38.0	+ 7.1	+ 19.6	+ 4.5

members, and the maximum and minimum stresses in the others. These tables indicate that two members of the lower chord are subject to both tension and compression, and must be designed accordingly.

Before measuring the stresses in the struts or posts it is desirable to prepare a table showing which diagonals are acting under the six possible combinations of load. Such a table for the ties FG and $F'G'$ is shown below, the acting tie being indicated by an asterisk. It being remembered that no counters act in any other panels, it is apparent that the stresses to be obtained from the diagrams for the posts of the counterbraced panels are those given in the first one of the

	EF			GH			G′H′			E′F′		
Dead load..........	m.	− 3.0	m.	m.	− 3.8	m.	m.	− 3.8	m.	m.	− 3.0	m.
	m.	− 2.2	c.	c.	− 3.1	m.	m.	− 3.1	c.	c.	− 2.2	m.
Snow load..........	m.	− 2.4	m.	m.	− 3.1	m.	m.	− 3.1	m.	m.	− 2.4	m.
	m.	− 1.6	c.	c.	− 2.5	m.	m.	− 2.5	c.	c.	− 1.6	m.
Wind on fixed side..	m.	− 6.7	m.	m.	−15.6	m.	m.	0	c.	c.	− 3.5	m.
Wind on free side...	m.	− 5.2	c.	c.	0	m.	m.	−14.2	m.	m.	− 6.7	m.

	FG		F′G′		EF	GH	G′H′	E′F′
	m.	c.	m.	c.				
Dead load	*		*		− 3.0	− 3.8	− 3.8	− 3.0
Dead + snow loads.......	*		*		− 5.4	− 6.9	− 6.9	− 5.4
Dead + wind on fixed side	*			*	− 9.7	−19.4	− 3.1	− 5.7
Dead + snow+wind fixed	*			*	−12.1	−22.5	− 5.6	− 7.3
Dead + wind on free side		*	*		− 7.4	− 3.1	−18.0	− 9.7
Dead + snow + wind free		*	*		− 9.0	− 5.6	−21.1	−12.1
		Maximum..............			−12.1	−22.5	−21.1	−12.1
		Minimum....			− 3.0	− 3.1	− 3.1	− 3.0

following tables. The ties which act on each side of the posts are indicated by *m* and *c* for the main and counter diagonals respectively. In some trusses three or four stresses may be required in some of the posts for the dead and snow loads and two for the wind loads. From this data the second portion of the last table may now be filled out, and the maximum and minimum stresses selected by inspection. It will be observed that the final values for EF are the same as for $E'F'$, and that for GH and $G'H'$ the minimum stresses are equal, while the maximum ones differ but slightly.

If the counter FG were omitted, and the main tie FG were then replaced by a member which could take both tension and compression, it would change the minimum stress in GH to $+ 0.4$, while if the main tie were omitted and the counter replaced by a counterbraced member it would change the maximum in GH to $- 12.7$, the maximum in EF to $- 9.0$, and the minimum in EF to $+ 1.8$ thousand pounds. It would seem therefore that it would be preferable to counterbrace the panels in this case instead of the members that would otherwise require it.

The following table gives the stresses in the remaining posts and in the middle suspender.

	AB	AB'	CD	$C'D'$	IJ	$I'J'$	KK'
Maximum........	− 65.4	− 48.6	− 37.2	− 31.5	− 44.6	− 35.2	+ 11.5
Minimum........	− 16.5	− 22.0	− 12.0	− 13.4	− 3.7	− 10.9	+ 1.3

Prob. 75. Find the maximum and minimum stresses in the crescent roof-truss treated in Art. 24, provided the panels be counterbraced with diagonals which take compression only.

ART. 64. A FERRIS WHEEL WITH TENSILE SPOKES.

The skeleton diagram of a small Ferris wheel with eight apexes and supported at the hub is given in Fig. 126. The broken circular line indicates the rack where the power is applied to rotate the wheel. The spokes are designed to take only tension, and equal loads are applied at all the apexes. The crown of the wheel tends to sag under the influence of the load at apex 1; but as this would produce compression in the spoke *IB* it will not act, and therefore the load is sup-

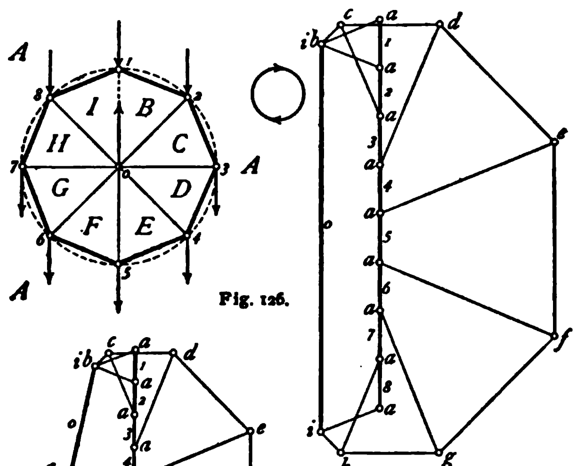


Fig. 126.

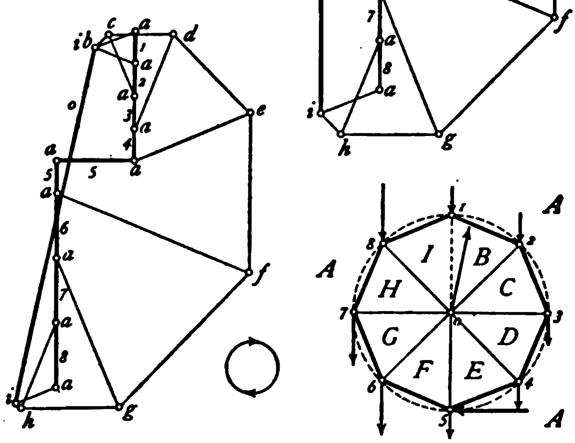


Fig. 127.

ported by the segments *AI* and *AB* of the rim. These stresses are obtained by constructing the force polygon for the apex 1 as shown in the stress diagram at the right. The force polygons for the remaining apexes may now be constructed in regular order. The long vertical *ib* is the reaction of the support and the diagram shows that it is in equilibrium with the stresses in all the spokes, which truly expresses the relation of the forces at the hub. In this article the resistance due to friction in the bearings is not taken into account.

An examination of this diagram shows that under uniform load the stresses in the spokes and in the segments of the rim gradually increase from the top to the bottom of the wheel, those in the rim being compression throughout. When a segment of the rim is in a horizontal position at the top of the wheel its stress is $-1.0P$, P being the apex load. The stress diagram for this position is not given. As it revolves its stress gradually increases to the maximum value of $-3.92P$ when the segment reaches the position of AE, the stress now remains unchanged until the position of AF is reached, and then gradually diminishes until the segment is again horizontal at the top. If the wheel had 16 spokes the compression in any segment of the rim would vary between the limits $2.414P$ and $7.689P$.

The stress in any spoke remains zero during the interval between its two upper positions which make an angle with the vertical equal to one-half the angle between the spokes, and then gradually increases until it reaches its maximum value of $+4P$, when the spoke is below the hub in a vertical position. It is interesting to observe that the maximum stress in any spoke is independent of the number of spokes when that number is not less than four. The construction of the diagram also indicates that the stresses in both rim and spokes are independent of the size of the wheel, except so far as the apex loads may depend upon it.

Fig. 127 gives the wheel and stress diagrams for the case when only three of the observation cars are occupied. The horizontal reaction applied on the circular rack which is used to rotate the wheel is found by equating to zero the sum of the moments of all the external forces with reference to the centre of rotation of the wheel. In the stress diagram the points *i* and *b* which coincide indicate no stress in the vertical spoke *IB*, while the inclined line *ib* is the reaction *IB* of the supports, for, since the spoke has no stress the letter *I* really applies to the entire space on the left of the inclined arrow designating the reaction. It will be noticed that most of the stresses on the loaded side are considerably greater than those on the other.

The Ferris wheel at the World's Columbian Exposition in 1893 was 250 feet in diameter and had 36 spokes. An article on this subject applying the graphic method to water wheels of a given type as well as to other structures, and including a description of the main features of a wheel of the same magnitude as that at the Exposition, but designed so that its stresses should be statically determinate, may be found in Zeitschrift für Bauwesen, Vol. XLIV, page 586 (1894).

Prob. 76. Determine the stresses in all the members of a Ferris wheel with 8 tensile spokes for a load *P* at each apex, when one of the spokes is vertical, and also when one of the rim segments is horizontal.

ART. 65. A BICYCLE WHEEL WITH TENSILE SPOKES.

Bicycle wheels are usually constructed with spokes of very light steel rods and stiff rims of metal or wood. The number of spokes is generally 32. A wheel with one-half that number of spokes is represented in Fig. 128. It carries a load *W* at the hub, and is subject to an equal reaction at the point where it rests upon the ground.

The reaction *BC* tends to produce compression in the spoke attached to the same point of the rim, but as it can take only tension it will not act, and hence may be considered as removed for the time being. The force triangle *abc* accordingly expresses the condition of equilibrium at apex 1 between the reaction *BC* and the direct stresses in *AB* and *AC*. The sides *ab* and *ac* are parallel to the chords of the arcs *AB* and

AC respectively. These stresses are equal, and their value is $-2.563 W$. The complete stress diagram is found to have a form similar to that of the wheel, except that the rim is composed of straight segments, and shows that the tension in all the spokes except *BC* is *W*, while the compression in all the segments of the rim is $2.563 W$. The perimeter of the stress

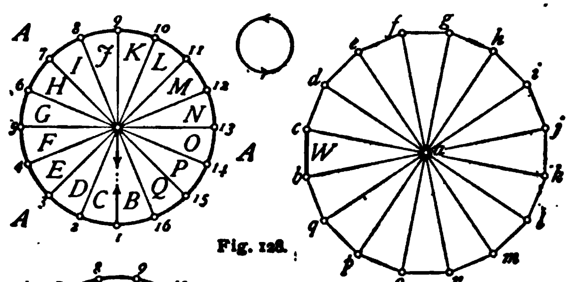


Fig. 128.

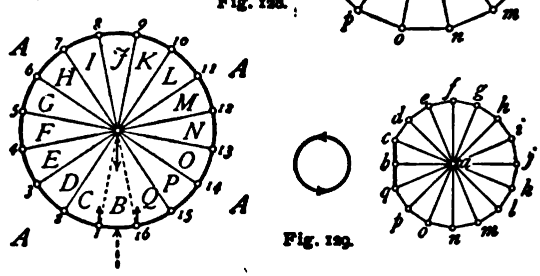


Fig. 129.

diagram expresses the condition of equilibrium at the hub between the load and the stresses in all the spokes.

The direct compression in any segment causes flexure, the fibers on its inner side being in compression, and those on the outside in tension. When the wheel rests on the ground at any intermediate points of a segment of the rim, the segment serves as a beam to transfer the reaction to the adjacent apexes or panel points of the wheel truss, the resulting flexure causing tension in the inner fibers and compression in the outer ones. Fig. 129 gives the position when the support is midway between the apexes 1 and 16. The direct stresses in the wheel members may therefore be found by replacing the reaction by the two upward forces at 1 and 16, each equal to ½ W. The stress diagram is readily constructed, as both spokes QB and BC are not acting. The tension in the remaining spokes is 0.510W, and the compression in all the segments of the rim except AB is 1.307 W, that in AB being 1.207W.

It is seen, therefore, that in passing from the position in Fig. 128 to that in Fig. 129 the direct stresses in the entire rim and in all the spokes except two, are reduced nearly one-half, and during the next thirty-second of a revolution the stresses increase again to their former value. This cycle of changes occurs 16 times in every revolution for each segment of the rim and 14 times for the spokes. In addition to this the stress in each spoke changes from W to 0 and back again to W in passing from the position of QB to that of CD in Fig. 128. Further, the bending moment due to the reaction of the ground, as well as that due to the direct stress in each segment of the rim, passes through a similar cycle of changes 16 times in each revolution. The form of the stress diagram shows that if the number of spokes is doubled the direct stress in the rim segments is almost doubled, while the bending moment is reduced nearly 75 per cent. On the other hand, the magnitude of the stress in the spokes is independent of their number.

The changes in stress caused by attaching the spokes, in the customary manner, in series tangent to opposite sides of a hub of given diameter instead of radiating to its center, can readily be found by applying principles heretofore given.*

Prob. 77. Find the stresses in a bicycle wheel with 32 spokes for both positions given in Figs. 128 and 129 when the load W is 150 pounds.

* See Eng. News, v. 38, pp. 100, 138, Aug. 12, 26, 1897.

CHAPTER VII.

ELASTIC DEFORMATION OF TRUSSES.

ART. 66. THE DISPLACEMENT DIAGRAM.

The change in length λ of any member of a truss which is subject to given loads and reactions may be computed by the well-known formula (Mechanics of Materials, Art. 5)

$$\lambda = \frac{Pl}{AE},$$

in which l is the length of the given member, A the area of its cross-section, E the coefficient of elasticity of the material of which it is composed, and P the total stress in the member. In case stresses due to temperature are to be taken into account the above value of λ must be combined with the quantity αtl, in which α is the coefficient of linear expansion for a change of one degree, and t the rise or fall of temperature expressed in degrees.

As a truss is composed of triangles, the method of finding the displacement of its panel points due to any given loads may be illustrated by showing how to determine the displacement of one panel point when two others with which it is connected by truss members are known. Let the panel point c in Fig. 130 be connected with a and b by members whose lengths are l_1 and l_2 respectively. Let the stress in ac be a compression which produces a shortening of λ_1 in its length, while the stress in bc is tension and λ_2 is the corresponding elongation. The magnitudes and directions of the displacements of a and b are represented by the lines aa' and bb'.

Let $a'c_1$ be drawn parallel and equal to ac. Since the stress in ac is compression, λ_1 must be laid off from c_1 towards the point a', which for the moment is to be regarded as fixed. This shortening, indicated by a heavy full line, is very much exaggerated in the figure, for if it were laid off to the same scale as $a'c_1$ it would not be visible. With a' as a center and the reduced length $l_1 - \lambda_1$ as a radius let an arc be described.

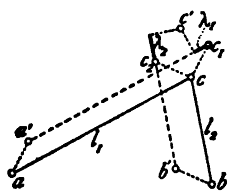


Fig. 130.

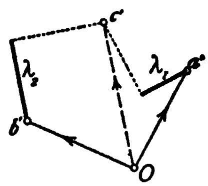


Fig. 131.

The panel point c must lie somewhere on this arc. Because the elastic deformations of the truss members are, however, very small, the tangent to the arc may be substituted for the arc itself. A perpendicular to $a'c_1$ is therefore drawn at the end of the line marked λ_1. Similarly, $b'c_2$ is drawn parallel to bc, and its length increased by its elongation λ_2 and a perpendicular erected at its extremity. The point c' is therefore at the intersection of these two perpendiculars, and the line cc' (not drawn) represents the displacement of c in magnitude and direction.

In view of the exceedingly small values of λ as compared with l it is desirable to exclude from the diagram that portion which contains the lines representing the lengths of the members themselves. This can readily be done, as it is seen that the lines cc_1 and cc_2 representing the displacements of a and b, the lines λ_1 and λ_2, and the perpendiculars at the extremities of λ_1 and λ_2 form a closed polygon. In Fig. 131 it is drawn separately to thrice the scale of that in Fig. 130, the pole O in the former replacing the point c in the latter. The displacements of the panel points *a*, *b*, and *c* all radiate from the pole *O*.

Such a diagram is called a displacement diagram. In its construction especial care must be exercised in observing the directions in which the values of λ are to be laid off, constantly referring to the panel points of the truss diagram, which for the time being are considered as fixed. The lengths of the perpendiculars whose intersections locate the successive panel points need not be measured.

Prob. 78. A weight of 15 000 pounds is suspended from a ceiling at points 10 feet apart by means of two wrought-iron bars, one being one inch square and 9 feet long, and the other 3/4 inch square and 10 feet long. Find the displacement of the point where the weight is attached to the bars.

ART. 67. DEFORMATION OF A TRUSS.

It is required to find the displacements of the panel points of a wooden king-post truss whose span is 16 feet and depth 8 feet, which carries a load of 12 000 pounds at panel point *b* (Fig. 132). The data required for the construction of the displacement diagram is given in the following table. In computing λ the value of E was assumed 1 500 000 pounds.

MEMBER.	STRESS.	LENGTH.	CROSS-SECTION.	λ	MEMBER.
	Pounds.	Inches.	Square Inches.	Inches.	Number.
$ab = bc$	+ 6 000	96	36	+ 0.0107	1 and 5
$aB = Bc$	− 8 490	135.75	64	− 0.0120	2 and 4
Bb	+ 12 000	96	36	+ 0.0213	3

For the sake of illustration let the point *a* be fixed, and the point *c* be regarded as perfectly free to move horizontally, although in practice such a short span is fixed at both supports, since the horizontal movement of *c* due to the load is too small to require a movable support.

The displacement diagram may be constructed by beginning at any panel point and regarding as fixed its own position as well as the direction of one member attached to it. Let the point a (which is actually fixed) and the direction of ab be so

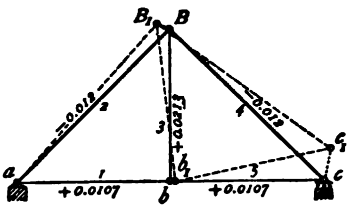


Fig. 132.

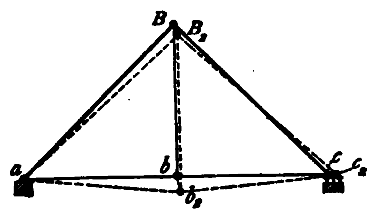


Fig. 133.

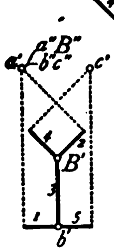


Fig. 136.

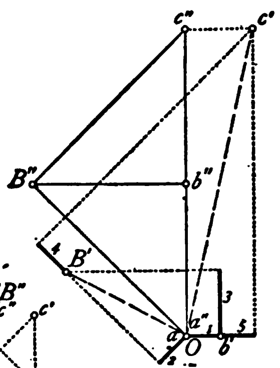


Fig. 134.

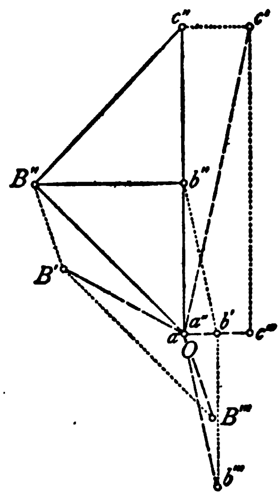


Fig. 135.

regarded. In Fig. 134 the point a' will therefore coincide with the pole O and λ_1 will be laid off toward the right, that is, in the direction of a toward b on the truss diagram. For convenience the values of λ are marked on the truss diagram, and when they are laid off in Fig. 134 they are marked by the same numbers as the corresponding members in Fig. 132. After b' is thus determined the displacement of B is next found by regarding a and b in the triangle aBb as fixed. The elongation λ_8 is laid off upward from b', and the shortening λ_9 downward from a', the intersection of the perpendiculars giving B'. In the same way c' is located. The lines OB' and Oc' are the displacements of B and c.

In order to show the deformed truss under the conditions assumed (that a and the direction of ab are fixed), the displacements are laid off on Fig. 132 to one-tenth of the scale employed in Fig. 134, and the corresponding points joined by broken lines. The student will observe that the deformation shown is greatly exaggerated, and hence the members seem to have unduly altered their lengths.

The primary conditions of the problem, however, require that c shall move only in a horizontal line, and therefore the entire truss must be revolved about a as a center until c_1 falls into the horizontal through c. As the arc thus described is very small compared with the radius ac_1 which in turn differs but very little from ac, a perpendicular to ac from c_1 may be substituted for the arc without appreciable error.

In Fig. 135, to which were transferred the displacements Ob', OB', and Oc' without the construction lines, the corresponding path of rotation of c is represented by $c'c'''$ which is drawn perpendicular to ac in Fig. 132, and continued until it meets the line Oc''', which is drawn parallel to the direction in which the panel point c is free to move. In this example that direction is horizontal, and happens to coincide with the line ac, but the statement here given is so framed as to apply equally to inclined lines of motion of panel points supported by expansion rollers or rockers. When the successive displacements Oc' and $c'c'''$ are combined, the resultant displacement is Oc'''. The displacement of B due to the rotation of the truss is $B'B'''$, which is perpendicular to aB (in Fig. 132), and whose length is proportional to its distance from the center a. That is, $B'B''' : c'c''' = aB : ac$, from which the length of $B'B'''$ may be conveniently found by similar triangles. Similarly, as ab equals one-half of ac, $b'b'''$ equals one-half of $c'c'''$. The resultant displacements are then represented in direction and magnitude by OB''', Ob''', and Oc''', and as a is the center of rotation, a''' coincides with a' and Oa''' is zero.

In Fig. 133 the final position of the deformed truss is shown in broken lines, the resultant displacements BB_2, bb_2, and cc_2 being laid off parallel to and equal to one-tenth of the lengths of OB''', Ob''', and Oc''' in Fig. 135. If the deformation were not exaggerated the truss diagram $aB_2c_2b_2$ in Fig. 133 would be equal to $aB_1c_1b_1$ in Fig. 132 in both form and dimensions.

For the purpose of simplifying the construction let the three parallelograms in Fig. 135 be completed, and the points a'', B'', c'', and b'' joined by lines as indicated. The lines $B''a''$, $b''a''$, and $c''a''$ (parallel and equal to $B'B'''$, $b'b'''$, and $c'c'''$) represent the displacements of panel points B, b, and c due to rotation about a, and are respectively perpendicular and proportional to aB, ab, and ac of the truss diagram, and therefore it follows that the diagram $a''B''c''b''$ is similar to $aBcb$, and all of their lines are mutually perpendicular. This important fact furnishes a means of determining the final displacements on Fig. 134 in a very simple manner, as follows: Through c' draw $c'c''$ parallel to the constrained line of motion of the panel point c, and draw $a''c''$ perpendicular to ac (in Fig. 132), and intersecting $c'c''$ at c''. On $a''c''$ draw a diagram similar to the truss diagram. The required displacements are then given by the directions and distances of the points B', b', and c' from B'', b'', and c'' respectively. It is thus seen that the points B'', b'', and c'' in Fig. 134 correspond to what may be regarded as successive positions of the shifted pole O in Fig. 135, which conception aids the memory in reading the directions correctly.

It is very desirable in practice to reduce the displacement diagrams to their most compact form. It will both diminish the errors due to slight inaccuracies in the directions of the intersecting perpendiculars as well as allow increased precision by the use of a larger scale. This result may be secured by beginning the construction with the line which suffers the minimum change in direction under the influence of the given loads. In simple trusses some line may be found at the middle which does not change its direction at all, provided the truss and the loading are both symmetrical with reference to a vertical section at the middle, or which changes but little in unsymmetrical trusses. For a bridge truss with an even number of panels, the middle vertical is such a member, or the chords of the middle panel when the number of panels is odd. As an illustration of the effect thus produced, the displacement diagram for the above truss when the direction of the middle vertical and the position of either of its extremities is assumed to be fixed, is given in Fig. 136 to the same scale as that in Fig. 134. Since a perpendicular to *ac* through *a'* meets a horizontal through *c'* at *a'*, the diagram *a''B''c''b''* is thereby reduced to zero. If the scale were doubled the vertical dimension of this diagram would not be quite equal to that of the one previously drawn. With a larger number of panels the difference is still greater. In this case the diagram makes a direct comparison between the displacements of all the panel points.

On applying the scale and protractor to the original drawing the displacement of *B* and *b* were found to be 0.0296 and 0.0501 inches, their directions being inclined 12¼° and 21¼° to the vertical. The angles were read only to the nearest quarter degree. As the lower chord is horizontal, the displacement of *c* is the sum of the elongations λ_1 and λ_2 which equals 0.0214 inch.

In order to avoid the excessive labor of making corrections in the cross-sections of tension members to allow for the effect of rivet holes in riveted shapes or of mortices or other cuts in timber, it is customary to use the gross sectional areas and to reduce somewhat the coefficient of elasticity.

Prob. 79. A single-track through Pratt truss railroad bridge has 5 panels, each 23.1 feet long and 25 feet deep. Using the same notation as in Fig. 96 in Art. 45, the members have the following areas of cross-section in square inches: $BC = CD$, 27.99; $ab = bc$, 16.44; cd, 17.62; aB, 29.75; Bb, 11.58; Bc, 10.5; Cc, 11.58; $Cd = cD$, 8.04. The tie Bc and the chord cd are composed of eye-bars of medium steel, and the rest of the members are built up with soft steel shapes. Find the displacements of the panel points of the lower chord due to a live load of 4000 pounds per linear foot combined with two excess loads of 26 000 pounds two panel lengths apart. (Use values of E of 29 000 000 and 26 000 000 pounds for the medium and soft steel.)

ART. 68. DEFLECTION OF A TRUSS.

While the displacement diagram gives the actual displacements of the panel points in the plane of the truss, their vertical components only are generally required. When bridge trusses are erected they are cambered so that under their maximum load the panel points of the loaded chord shall not fall below a horizontal line joining the panel points at the supports. This camber is secured by shortening the tension members by an amount equal to the elastic elongation due to the sum of the live and dead load stresses, when the live load is so placed as to produce the maximum moment at the middle panel point, plus an allowance for clearance in the case of pin-connected joints. The compression members are lengthened in a similar way. (See Part III, Art. 62.) The maximum stresses must not be employed throughout because they are not simultaneous.

If to these changes of length for the members there be added the values of λ caused by the dead load only, due regard being paid to their respective signs, and the corresponding displacement diagram be drawn, the vertical components of the displacements will give the deflections of the several panel points when the bridge supports only the dead load, and their values may be used for comparison with the observed deflections. Roof trusses supporting horizontal ceilings are cambered in a similar manner.

In the example used in the preceding article, the deflection of b is found to be 0.0489 and of B is 0.0276 inch. The diagram shows that when two points are directly above each other their deflections should differ by the change in length of the member connecting them. This may serve as a useful test of the accuracy of the drawing.

Prob. 80. Find the deflection due to the live load, of the bridge whose data are given in Prob. 79.

ART. 69. TRUSS DEFLECTION UNDER LOCOMOTIVE WHEEL LOADS.

The most convenient way to find the stresses in the truss members whether the chords are both horizontal, or either one or both of them are arched, is the following: Let the position of the live load be found which produces the maximum moment at the panel point at, or nearest to, the middle of the truss (Art. 47 or 51) and with the truss diagram in this position on the equilibrium polygon let the closing line be drawn as well as the chords of the polygon whose horizontal projections equal the successive panel lengths in magnitude and position. The extremities of these chords connect the points two and two where the verticals of the truss diagram intersect the equilibrium polygon. By drawing rays parallel to these chords through the pole they will cut off on the vertical load line the panel loads for this position of the wheel loads, and a ray parallel to the closing line will divide the reactions. The stress diagram, which is similar to that for dead load, may now be completed in the usual manner.

The following table gives the required data for determining the deflection of the double-track through bridge No. 77 of the second division of the Baltimore and Ohio Railroad, due

MEMBERS.	STRESS.	LENGTH.	CROSS-SECTION.	λ	MEMBER.
	1000 lbs.	Inches.	Square Inches.	Inches.	Number.
$ab = bc$	+ 287.0	321	36.0	+ 0.0882	11, 8
cd	+ 360.0	321	46.0	+ 0.0866	4
de	+ 356.0	321	46.0	+ 0.0856	14
$ef = fg$	+ 266.5	321	36.0	+ 0.0819	18, 21
BC	− 369.0	329	65.12	− 0.0717	6
$CD = DE$	− 414.0	321	71.52	− 0.0715	2, 12
EF	− 365.0	329	65.12	− 0.0709	16
aB	− 413.0	464.7	76.44	− 0.0966	10
Bb	+ 140.0	336	17.52	+ 0.1033	9
Bc	+ 106.0	464.7	23.56	+ 0.0804	7
Cc	+ 12.5	408	30.6	+ 0.0064	5
Cd	+ 87.5	519.1	34.0	+ 0.0514	3
Dd	0	408	20.6	0	1
dE	+ 94.5	519.1	34.0	+ 0.0555	13
Ee	+ 5.0	408	30.6	+ 0.0024	15
eF	+ 130.0	464.7	23.56	+ 0.0986	17
Ff	+ 106.0	336	17.52	+ 0.0782	19
Fg	− 386.5	464.7	76.44	− 0.0903	20

to the specified live load of two B. & O. typical consolidation locomotives and train. The form of the truss is shown in Fig. 137. The lower chord consists of eye-bars of medium open-hearth steel, while the upper chord and web members are built up of shapes of soft steel. There are no counters. The stresses were found in the manner described above, the stress diagram being drawn to a scale of 50 000 pounds to the inch. The value of the coefficient of elasticity was assumed as 26 000 000 for soft and 29 000 000 pounds for the medium steel.

The displacement diagram, shown in reduced size in Fig. 138, was constructed by assuming the position of d and the direction of dD as fixed. It is the best one that can be drawn since dD changes its direction actually less than any other member, and requires very little rotation of the truss. The truss members are numbered in the order in which their values of λ were used in constructing the diagram. As the left end of the truss is fixed and the right end rests on expansion rollers, a'' coincides with a', and g'' lies in a horizontal through g'

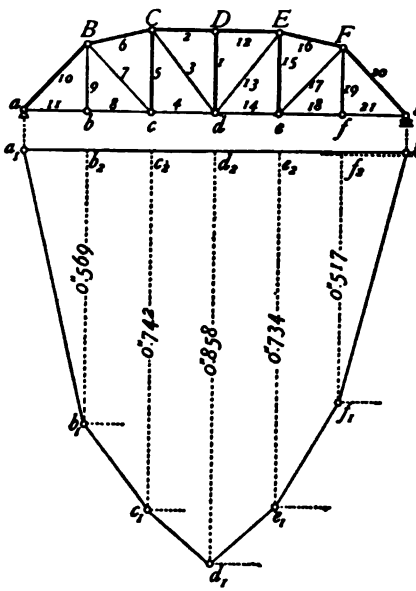


Fig. 137.

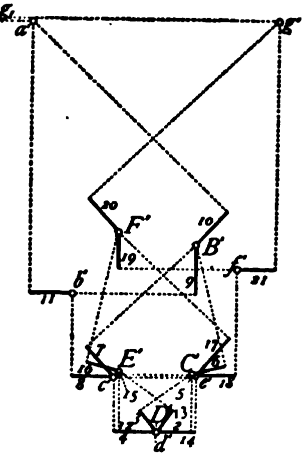


Fig. 138.

directly above a'. The distance $a''g''$ is too small a span to permit a diagram similar to the truss diagram to be drawn without confusion of points. As, however, only the deflections are desired, the necessity for this diagram may be obviated by the following construction of a deflection polygon.

Let a_1 be obtained by projecting a' across on the vertical through a, and similarly for g_1. The intersection of the line joining a_1 and g_1 with any vertical, as for example that through c, gives a point c_2 whose height is the same as c'' would have been if the diagram $a'' \ldots g''$ had been drawn. By projecting $b'c'd'e'$ and f' on the corresponding verticals and joining them as indicated, a polygon will be obtained whose ordinates at the panel points represent the corresponding deflections of the panel points of the lower chord. The values of the deflections in inches are marked on the diagram. The scale of the original displacement diagram was 0.060 inch to an inch. A deflection polygon for the panel points of the upper chord might also be drawn, if desired; but as these points are united to the lower chord by verticals, their deflections may be obtained by subtracting the elongations of the verticals from the corresponding deflections marked on the diagram.

Prob. 81. What change in the deflection would be caused in the truss used in this article by substituting medium steel eye-bars for *Bc* and *Cd* with sectional areas of 24.0 and 20.25 square inches respectively?

ART. 70. SPECIAL CONSTRUCTION FOR CENTER PANEL.

When a simple truss has a center panel and the loading is symmetrical both of the diagonals in that panel have no stress and hence no linear deformation. If either one of the diagonals is omitted the resulting deflection polygon is not quite symmetrical. If the other diagonal is omitted the deflections of corresponding panel points are interchanged and hence the true deflection of each panel point is the mean of these two values. The necessity of constructing a displacement diagram for more than half of the truss may be avoided by means of the following expedient: Let an imaginary vertical be inserted in the middle of the center panel, with a shortening equal to that of the vertical on each side of the panel. The vertical will not change its direction when the truss deflects under symmetrical loading. Hence the displacement diagram is drawn by taking the direction of this vertical as fixed. If the diagram were drawn for the entire truss it would be symmetrical and the perpendicular truss diagram (which represents the rotation of the given truss, if required) would be reduced to a point as in Fig. 136 of Art. 67. Hence the displacement diagram for only one-half of the truss is required.

In Fig. 139 the imaginary vertical *Mm* joins the middle points of the upper and lower chord members of the center panel. Taking *m* and the direction of *Mm* as fixed, the point *m′* in Fig. 140 is located in any convenient position. From *m′* the deformation of *Mm* (equal to that of *Ff*) is laid off parallel to *Mm*. It is laid off downward since *M* moves downward toward the fixed end *m*, when the member is shortened.

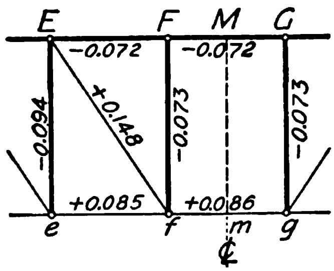


Fig. 139.

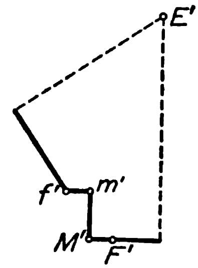


Fig. 140.

Moreover, since *FG* does not change its direction when the truss deflects, the point *F′* may be located directly by laying off toward the right from *M* and parallel to *FM* (or *FG*) one-half of the deformation of the chord *FG*. Since *M′* was located before *F′*, *M* is regarded now as fixed; as the member is shortened *F* moves toward the right and hence the change of length is laid off toward the right from *M′*. For the same reason *m′f′* is drawn parallel to *fm* (or *fg*) and equal to one-half of the elongation of *fg*. It is laid off toward the left as *f* moves toward the left from the fixed end *m* when the chord elongates. The rest of the construction is the same as was fully explained in Art. 67, only a small portion of which is shown in Fig. 140.

Prob. 82. Referring to Fig. 46 in Art. 25, which member must be taken as fixed in direction in order to secure a symmetrical displacement diagram? Must the entire diagram be constructed?

CHAPTER VIII.

INFLUENCE LINES FOR STRESSES.

ART. 71. THEORETIC RELATIONS.

An influence line is a line which shows the variation of a reaction, vertical shear, bending moment, stress, or any other function, when a single concentrated load moves across the span of a beam or truss. The variation of the reaction of one of the supports of a beam or bridge, as a single load P crosses from one end to the other, may be exhibited by a line called 'a reaction influence line.' To draw it, the values of the reaction for several positions of the load P are laid off as ordinates at those positions, and the line joining their tops is the desired influence line. For example, let it be required to draw the influence line for the right reaction of the simple beam shown in Fig. 141, if z is the distance of P from the left support and l is the span, the right reaction $R_2 = Pz/l$ and the value of this is P when the load is at the right support, $\frac{1}{2}P$ when the load is at the middle of the span, and zero when the load is at the left support. The line joining the tops of the ordinates is a straight line as shown in the figure, each ordinate giving the value of the reaction R_2 for a load P directly above it.

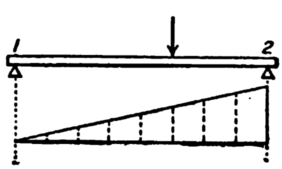


Fig. 141.

The variation of the vertical shear at any given section of a beam or bridge, as a single load P crosses from one end to the other, may be exhibited by a line called 'the shear influence line.' To draw it the values of the vertical shear for several positions of the load are laid off as ordinates at those positions. For example, the vertical shear at the section K of the simple beam in Fig. 142 due to a load P at the distance kl from the left support, is $+P(1-k)$ when the load is on KB, and $-Pk$ when the load is on AK. Hence when P is at A or B the ordinate is zero, when P is passing K the ordinates are $+P(1-k_1)$ and $-Pk_1$, the distance AK being k_1l. These lines clearly show how

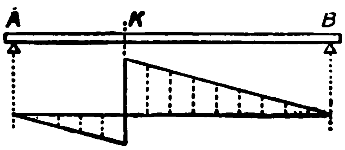


Fig. 142.

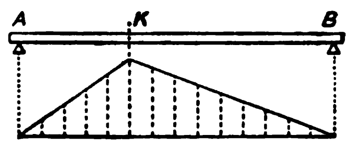


Fig. 143.

loads should be placed in order to give respectively the greatest positive and negative shears at the given section K.

The variation of the bending moment at a given section may also be represented by a line called 'the moment influence line.' Thus, for the section K at the distance k_1l from A, in the simple beam of Fig. 143, the moment due to a load P at a distance kl from A is $+Pk_1l(1-k)$ for a load on the right of K, and $+Pkl(1-k_1)$ for a load on the left of K. Since the bending moment at the given section varies as the first power of kl in both cases, the influence diagram consists of two straight lines, and these are readily drawn after erecting an ordinate at the given section to represent the moment $Pk_1l(1-k_1)$ due to a load at that point. This diagram shows that the maximum bending moment at any section of a simple beam occurs when the span is fully loaded, and when the heavy loads are near the section.

It is very important to distinguish clearly the difference between a 'bending moment diagram' and a 'moment influence diagram.' In the former, the load or loads are fixed in position and each ordinate represents the bending moment in a section of the beam having the same location as the ordinate. In the latter, the given section has a fixed location and each ordinate represents the bending moment in that section of the beam when a single concentrated load occupies the same position as the ordinate. Stated in another way, in the former, the diagram gives the bending moments in the different sections of the beam, for loading in one position; while in the latter, the diagram gives the bending moment in only one section of the beam for the different positions of a single moving load. Usually the moving load is taken as 1 kip or 1 pound in constructing influence lines.

The variation in the stress in any given member of a truss may be represented likewise by a line called 'a stress influence line.' It may also be constructed by finding the values of the stress due to several positions of the load P, laying them off as ordinates, and joining their tops as described previously for other kinds of influence lines. In general, for members in a simple truss, only two such ordinates are required, located at the ends of the panel which contains the member, the stress influence line being composed of three straight lines. Frequently, only one ordinate is required while the stress influence line is reduced to two straight lines.

Another method of constructing a stress influence line is by means of a deflection diagram for the truss due to a unit change

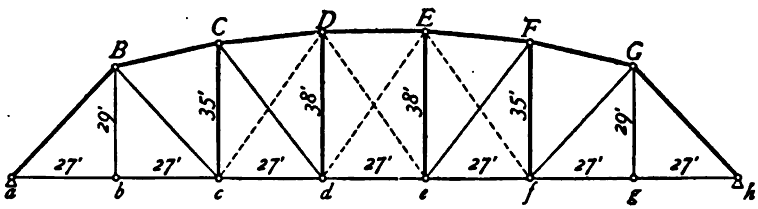


Fig. 144.

in length of the given member. Referring to the truss in Fig. 144 let a load P be applied at panel point e, and let Δ be the deflection of the truss at the same point. The load P causes stresses and hence changes of length in nearly all the members of the truss. The change in length of each member contributes a certain increment towards the total deflection Δ. For example, let the member *Bc* be taken. Let *S* be its stress due to the load *P*, let λ be its corresponding change in length or its linear deformation, and let δ be the increment of deflection at *e* due to the change in length of *Bc* only. By equating the external and internal work (see Roofs and Bridges, Part I, Art. 85) there is obtained the equation.

$$\tfrac{1}{2}P\delta=\tfrac{1}{2}S\lambda \quad \text{or} \quad P\delta=S\lambda.$$

Now if *P* be made equal to a unit load, and λ be made equal to a linear unit, the equation becomes

$$1\cdot\delta=1\cdot S,$$

which shows that the numerical magnitude of the deflection equals that of the stress *S*, provided each is expressed in the proper units. The same relation exists for any position of the load. It follows, therefore, that the deflection diagram of the truss due to a unit change in length of the given member is identical with the influence diagram for the stress in the member.

To construct the deflection diagram which is to be used as a stress influence diagram the given truss member is assumed to have a unit linear contraction, while all the other members remain unchanged in length. The member is assumed to be shortened instead of lengthened in order that positive ordinates, that is, those which are measured upward from the closing line or axis, may denote tension, while negative ordinates, or those measured downward from the closing line, may denote compression.

The deflections of the panel points of the truss are found by means of a displacement diagram as explained in Arts. 66, 67 and 68. The displacement diagram may be constructed by considering both the position of any joint of the truss and the direction of any member attached to it as temporarily fixed.

In order to make the diagram as simple as possible for members in the left half of the truss, it is best to commence at a joint at the right end of the panel in which the member is situated, and to regard the position of this joint and the direction of a vertical member attached to it as fixed. For members in the right half of the truss it is best to begin at a joint at the left end of the panel containing the member.

As explained in Art. 67 the final displacement of each joint of a truss is the resultant of two component displacements. The first of these is due to changes in the lengths of the truss members under an assumed condition that the position of a certain joint and the direction of a member attached to it are both fixed. Since this assumed condition often does not agree with the actual limitations of the movements of certain truss joints it becomes necessary to rotate the truss about a certain joint to make it conform to these limitations. The second component displacement of each joint is derived from this rotation of the truss. The graphic methods for determining these two sets of displacements were introduced respectively by WILLIOT and MOHR, and hence the combined displacement diagram is often called a WILLIOT-MOHR diagram.

The method of constructing stress influence lines by means of displacement diagrams as described in this article is absolutely general, applying to any bridge truss with constant or variable depth, and with any kind of web system. By its use the necessity of remembering many different rules for constructing influence lines for different chord or web members is, therefore, avoided. Stress influence lines may accordingly be drawn without previously requiring an analysis of stress relations in various members of a truss; thus making this method especially useful in determining the variations of stresses in the members of new or uncommon types of trusses.

The general method of developing the subject of influence lines from the viewpoint of their identity with deflection diagrams, and some applications of this principle were first published in an article on Influence Lines as Deflection Diagrams by D. B. STEINMAN in Engineering Record, vol. 74, page 648, Nov. 25, 1916.

Prob. 83. Construct the influence diagram for the chord member *ab* in Fig. 133 of Art. 67.

ART. 72. INFLUENCE LINES FOR A PARKER TRUSS.

In Fig. 145 are shown influence lines for the stresses in an upper chord member, lower chord member, main diagonal, counter diagonal and a vertical in a Parker truss, as well as for a vertical under a special condition imposed upon an adjacent diagonal. The Parker truss is the same one used as an example in Chap. V, its diagonals taking tension only. Its dimensions and loading are given in Art. 51.

The displacement diagram shown at the left of the stress influence line for *cd* is constructed by the method given in Art. 67 as follows: The member *cd* is assumed to be shortened a linear unit or 1 inch, while the lengths of all the other members remain unchanged. The position of panel point *d* and the direction of the vertical *dD* are assumed to be fixed. Since there is no deformation in *dD* the point D' on the displacement diagram will coincide with the point d', which was first located at any convenient position on the drawing paper. From D' the deformation in the member *DC* is laid off equal to zero and a line drawn at its extremity (also D') in a direction perpendicular to the member *DC*; while from d' the deformation of the member *dC* is laid off equal to zero and a line drawn at its extremity (also d') in a direction perpendicular to *dC*; the intersection of these two perpendiculars gives the location of the point C'. In this particular case since D' coincides with d', the point C' also coincides with both. For the same reason E', e', F', f', G', g' and h' coincide with d'.

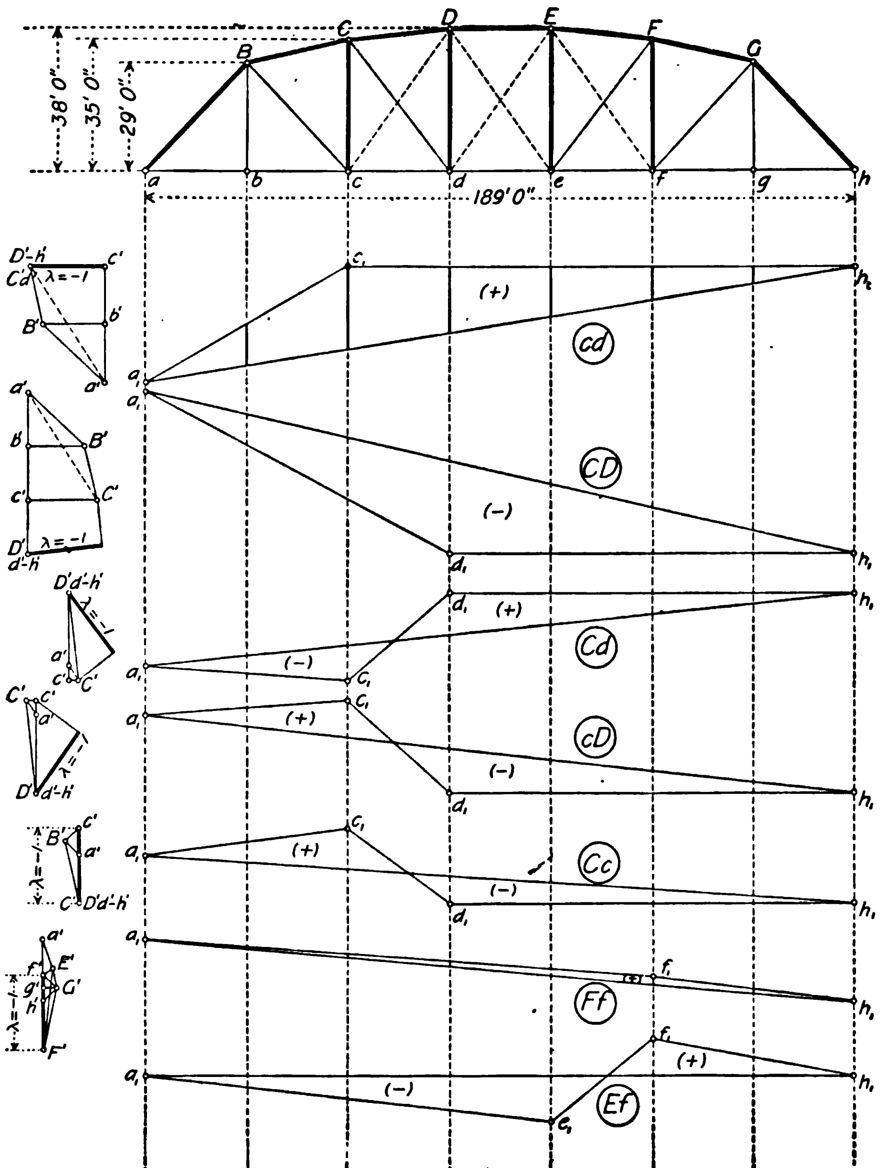


Fig. 145. Note.—In the first displacement diagram the line $B'c'$ should be inserted, and in the second one the lines $B'c'$ and $C'd'$.

To locate c', the deformations of Cc and of cd must be laid off in the proper directions, perpendiculars erected at their extremities and their intersections found. The zero deformation of Cc is laid off from C' and at its extremity (also C') a line is drawn perpendicular to the member Cc. Since the member cd is shortened, its left end c will move toward the right when its right end d is held in position, hence the deformation $\lambda = 1$ inch is laid off by scale to the right from d' in the displacement diagram and parallel to the truss member cd, and at its extremity a line is drawn perpendicular to it. The point c' is located at the intersection of both perpendiculars. At C' the zero deformation of BC is laid off and at its extremity a perpendicular to BC is drawn; from c' the zero deformation of Bc is laid off and a perpendicular to Bc is drawn; their intersection gives the point B'. In a similar manner the points b' and a' are located. It must be remembered that each perpendicular referred to above represents an arc of rotation for one end of a truss member about the other end whose corresponding point in the displacement diagram has been previously located (see Art. 66).

The deflection diagram may be drawn next by projecting the points a', b', c, . . . h' horizontally across to the corresponding verticals drawn through the panel points a, b, c, . . . h, of the truss, and joining these points as shown. The line a_1h_1 forms the closing line or axis of the diagram. This closing line takes the place of the auxiliary truss diagram whose members are respectively perpendicular to those of the given truss, when only vertical deflections are required. If the value of any deflection is desired the ordinate must be measured by the same scale as that used in laying off the shortening of the member cd. The ordinate below c measures 1.102 inches. According to the relation developed in the preceding article the deflection diagram is also the influence diagram for the stress in cd. If a load of 1 kip is applied at c the stress in cd is therefore 1.102 kips.

If there are several panels to the left of the one containing the member for which an influence diagram is to be constructed, a short cut may be employed to avoid locating all the corresponding points on the displacement diagram. For example, in the displacement diagram for the shortening of *cd*, after the points C' and c' are located, the point a' may be located next by considering the panel point *a* to be connected directly to *C* and *c* by two members *aC* and *ac*. These members have zero deformations since the members replaced by them had no changes in length. Hence $C'a'$ is drawn perpendicular to *Ca* and $c'a'$ perperpendicular to *ca*, thus locating a' by their intersection. If preferred, this method may be used to check the location of a' when determined by the method previously described.

In a similar manner, the influence lines for the members *CD*, *Cd*, *cD* and *Cc* are constructed. In the diagram for *Cd* the position of the point of division *o* is located where the line c_1d_1 crosses the closing line a_1h_1. The diagram also shows that any load on the left of the point of division causes compression provided there is no counter in the panel and *Cd* is designed to take both compression and tension. Similarly the diagram for *Cc* shows that any load on the left of its point of division causes tension in the member provided the main diagonal *Cd* is acting, since the displacement diagram was constructed for this condition.

The greatest stress in *Cc* occurs when the live load covers the portion of the span on the right of the point of division. For this loading the diagonal *Cd* is acting. All points of division should be checked by the method given in Art. 48.

The sixth influence diagram in Fig. 145 was drawn for the corresponding vertical *Ff* in the right half of the truss when the counter diagonal *Ef* is assumed to act. The displacement diagram was drawn by assuming the position of *f* and the direction of *Ff* to be fixed. It would have been a little simpler in form if *e* and the direction of *eE* had been assumed as fixed.

The line a_1f_1 would then be horizontal. The influence diagram shows that any load on the span causes tension in *Ff* under the condition assumed for its construction; that is, provided the diagonal *Ef* is acting, or when *Fe* is not acting. The influence line for *Ef* is placed directly below that for *Ff*.

If in any given case it is desired to secure a higher degree of precision than can be obtained from the measured ordinates of an influence line constructed by means of a displacement diagram, the stresses represented by the maximum positive and negative ordinates may be computed by the analytic method. For example, let it be required to find the value of the ordinate at f_1 for the vertical *Ff*. Referring to Fig. *b* on Plate V, as well as to the stress triangle at the upper chord U_5 in Fig. *c*, it will be seen that the most convenient method of computing the stress in the vertical when the counter acts on its left and the main diagonal on its right, is to find the ratio of its stress to the horizontal components of the stresses in the adjacent upper chords. This ratio is one-ninth, as may be seen by referring to Fig. 110 in Art. 55, in which the lengths of the lower chords and verticals are given. If a line be drawn through *E* parallel to *FG* it will intersect *Ff* at a distance of 3 feet below *F*, which is one-ninth of the panel length of 27 feet. For a load of 1 kip at *f* the left reaction is $\frac{2}{7}$ kip. Taking the moment of the reaction about *f* and dividing by the length of *Ff* the horizontal component of the stress in *EF* or *FG* is found to be $\frac{270}{245}$ kip. The stress in *Ff* is therefore one-ninth of this or $\frac{30}{245}=0.1225$ kip.

Prob. 84. Construct the influence diagram for the suspender *Bb* in Fig. 145.

ART. 73. STRESSES IN A PARKER TRUSS.

The value of any ordinate of a stress influence diagram for a given truss member when measured by the proper scale of loading gives the magnitude of the stress when a load of 1 kip or of 1 pound occupies the corresponding position. If the specified live load on the truss consists of equal panel loads, it is only necessary to measure all the ordinates below the loaded panel points, and to multiply the sum by the value of a panel load.

For example, let the dead load stress in *cd* be found from the influence line in Fig. 145. The ordinate at c_1 measures 1.102 kips. The sum of all the ordinates below the panel points is $3.5\times 1.102=3.857$ kips, and since the dead panel load is 29.7 kips the stress equals $29.7\times 3.857=+114.6$ kips. The value given on Fig. *b* of Plate V is +114.4 kips.

However, if the panel loads are unequal then the magnitude of each ordinate must be multiplied by the corresponding panel load, and the sum of these products obtained. Due regard must be paid to whether the ordinates are positive or negative.

When locomotive axle loads are specified it is not necessary to determine the panel loads for any given position of the live load, since the same result will be obtained by measuring the ordinate at the position of each axle load, multiplying its magnitude by the corresponding axle load and adding the products. Since locomotive axle loads are usually divided into several groups of equal loads, the number of products may be reduced by adding the values of the ordinates for each group of loads and multiplying the sum by the value of one of the axle loads in that group.

If a train load of 2 kips per linear foot covers a part of the span the corresponding stress is found by computing the partial area of the influence diagram for the given member, and multiplying this by 2.

The form of the influence diagram shows how to determine the position of axle loads graphically by stretching a thread when a tracing of the truss diagram is shifted over a stepped load line as illustrated in Fig. 100, Art. 49, for the diagonal *Cd*. The position of the thread is shown by the broken line *og* and it cuts the stepped load line at the point *i*. Referring now to the influence line in Fig. 145, the points o, i and g in Fig. 100 correspond in position to the points o, d_1 and h_1. The influence line for cd shows that the loads must cover the entire span, an axle load must be above the panel point c, and the thread which joins the two points of the stepped load line which are on the verticals through the end supports must cut the step above c, as illustrated in Fig. 90, Art. 42. The points a, i, and d in that figure correspond to a_1, c_1, and h_1 on the influence line. Whenever the influence line on one side of the closing line or axis forms a triangle the position of the axle loads may be similarly determined by stretching a thread.

Let it be required to find the largest tension in Ff which will be the minimum stress in Cc as well as in that of Ff. For a load of 1 kip at f the tension in Ff is $\frac{30}{245}=0.1225$ kip, which equals the measured value of the ordinate at f_1. The sum of all the ordinates under the panel points is 3.5 times the ordinate at f_1, and hence the dead load stress is $3.5 \times 1.3225 \times 29.7 = +12.7$ kips, the panel load being 29.7 kips. On Fig. b of Plate V, the stress is given as $+12.8$ kips. Since 8.1 kips of the total dead panel load is applied at the upper panel point, this stress must be corrected, making it $+12.7-8.1=+4.6$ kips.

The influence line for Ff shows that its greatest tension would occur when the live load covers the entire span, provided the counter Ef were acting, since this condition was assumed in constructing its influence line. As Ef, however, cannot take compression, the greatest tension in Ff will occur when the live load comes on from the right and extends just far enough to reduce the total stress in Ef (and also eF) to zero. By the method employed in Chap. V the required position was found to be that which placed axle 1 at a distance of 3 feet to the left of panel e, as stated in the fourth paragraph of Art. 54. For this position the stress in Ef is found as follows, by means of its influence line in Fig. 145. The ordinates at e_1 and f_1 are 0.5808 and 0.5007 kip respectively. The sums of the positive ordinates under the pilot, driver, and tender axles respectively are 0.280, 0.800 and 1.692 kips, and the stress due to these loads is $10\times0.280+20\times0.800+11.5\times1.692=+37.92$ kips. The negative ordinates under the pilot and driver axles are 0.563 and 0.556 kip, and the stress is $10\times0.563+20\times0.566=-16.75$ kips. The total live load stress is $+37.92-16.75=+21.5$ kips. The dead load stress obtained by the influence line is $(-1.452+0.751)29.7=-20.8$ kips. The combined live and dead load stress is therefore $+21.5-20.8=+0.7$ kip, showing that the stress in the counter is not reduced quite to zero. This position of the loads makes the stress in the vertical *Ff* equal to $10\times0.163+20\times0.5695+11.5\times0.4135=+17.78$ kips. Adding the dead load stress the total stress is $+17.8+4.6=+22.4$ kips. This checks exactly the combined stress of $+30.5$ kips, given in the fourth paragraph of Art. 54, before the correction of -8.1 kips was applied on account of the division of the dead panel load between the upper and lower panel points.

Now let the axle loads be moved one foot to the left. The live load stress in *Ef* becomes $+37.92-18.30=+19.6$ kips and the sum of the live and dead load stresses is $+19.6-20.8=-1.2$ kips; which shows that the axle loads should have been moved only about one-third of a foot. For this last position the live load stress in *Ff* is increased 0.08 kip, making the minimum stress in *Ff* (and in *Cc*) $+17.78+0.08+4.6=+22.5$ kips. A value of $+23.4$ kips was obtained in the latter part of Art. 54 for a different loading.

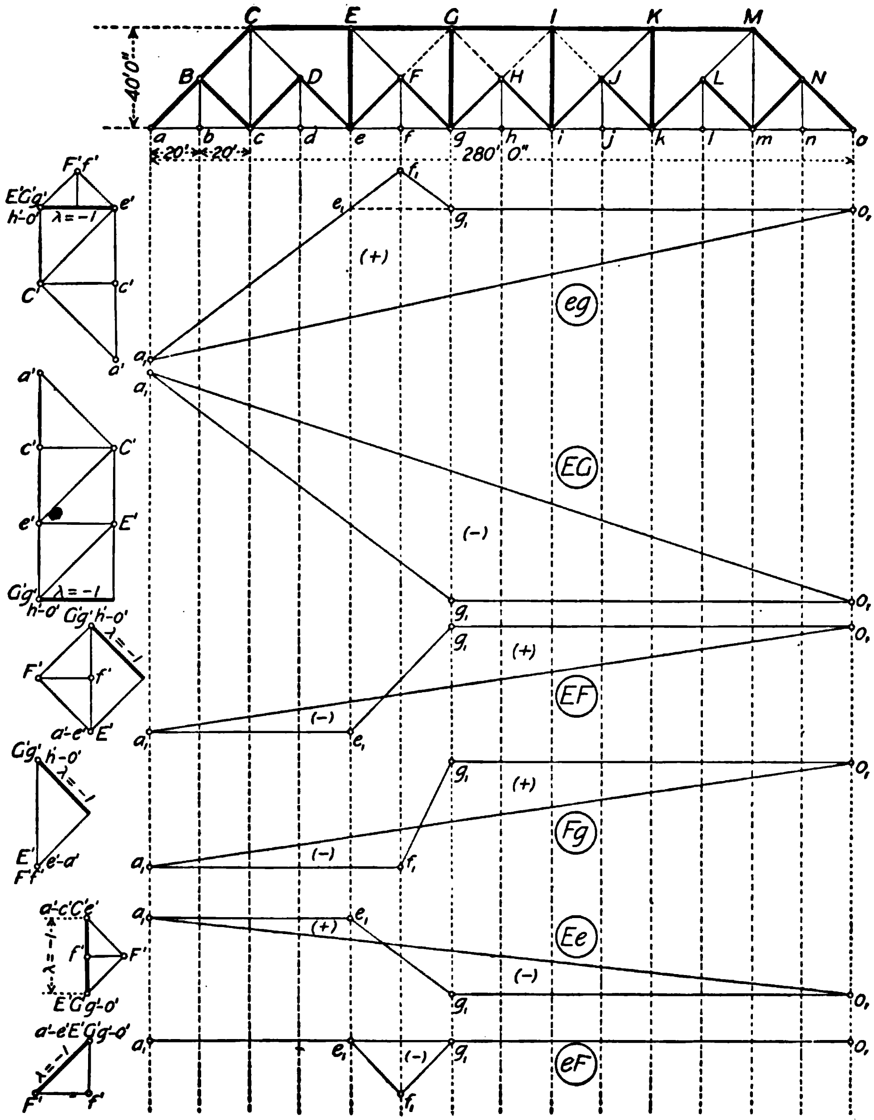
An approximate value of the minimum stress in *Ff* may be obtained as follows: Let a stress polygon be constructed for the members meeting at the joint *F* for an assumed live load compression in *eF* equal in magnitude to its dead load stress of $+19.7$ kips. This polygon gives a stress in *Ff* of $+33.6$ kips. The dead load stress in *Ff* when *eF* acts and the proper correction is made for a part of the panel load being applied at the upper panel point is $-4.2-8.1=-12.3$ kips. The combined stress is $+33.6-12.3=+21.3$ kips, which is 2.1 kips less than the correct minimum given in the table in Art. 54.

Prob. 85. By means of its influence line check the greatest tension in Ff for the position of the live load illustrated in Fig. 108, Art. 54, as well as the corresponding minimum stress.

ART. 74. INFLUENCE LINES FOR BALTIMORE TRUSS.

Fig. 146 shows the stress influence lines for most of the members in the sub-divided panel from e to g, and the displacement diagrams by means of which they are constructed. It will be observed that the influence diagrams for EG, EF and Ee are exactly the same as if the panels were not subdivided, or like those of a Pratt truss with 7 panels. That for Fg has the same form as if the truss were a Pratt truss with 14 panels. The influence line for eF has the same form as if the secondary truss eFg were acting independently. The influence line for the short suspender Ff has the same form as that for eF except that the vertical ordinate at f_1 measures 1 kip by the scale.

In constructing the displacement diagrams, the point g and the direction of Gg are assumed to be fixed as recommended in Art. 71. Since their general construction was explained in Art. 72, only those features are given special attention in this article which depend upon the effect of sub-divided panels. After locating the points g' and G' the points E' and e', representing panel points of a large or primary panel of the truss are to be located before F' and f' which represent panel points of the secondary truss. In the displacement diagram for EF it will be noted that since there is no deformation in Fg, the deformation in Eg is also $\lambda=-1$ inch, and hence E' is located as readily by laying off deformations from g' and G' and drawing perpendiculars at their extremities as though the intermediate panel point F did not exist. As indicated in Fig. 146 it is not necessary to locate the points b' and d' for the secondary trusses on the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eF
(+)
(-)
λ = -1

Fig. 146.

left of the panel containing the member whose linear deformation of 1 inch is laid off.

The influence diagram for *eg* shows the effect of sub-dividing the panel *eg*. If the line o_1g_1 is produced toward the left it intersects the line a_1f_1 at e_1, directly below the panel point *e*. Accordingly, the diagram may be regarded as a combination of the two triangles $a_1e_1o_1$ and $e_1f_1g_1$. The former is the influence line for the chord member *eg* when the panel *eg* is not sub-divided, while the latter is the influence line for *eg* as a member of the secondary truss *eFg*, thus indicating that the stresses for these two conditions may be obtained separately and added together, if desired. When locomotive axle loads are empolyed the same position of the loads must be used in both cases. A criterion for the correct position of the loads may be found by deducing a formula in a similar manner to that employed in Art. 45 of Part I. In this case it is desirable to use separate resultants for the loads from *a* to *e*, *e* to *f*, and *f* to *g*. The criterion is as follows:

Load from *a* to *f*−load from *f* to $g = Wl'/l$ in which *W* is the entire load on the truss, *l′* the distance from the left reaction to the center of moments, and *l* the span of the truss. For the analytic computations of the stress, the section would be passed vertically between *f* and *g*, hence the left-hand member of this equation may be expressed in more general terms as the load in the panels on the left of the section minus the loads in the panel cut by the section.

To satisfy this criterion the load must practically cover the entire span, an axle load being placed at *f*. It is also desirable to bring the heavier loads as near to *f* as possible while the preceding condition remains fulfilled. The graphic method of applying this criterion is as follows: when the tracing containing the truss diagram is placed over the sheet containing the stepped load line. Let a_2 be the intersection of the vertical at the left support *a* with the load line, and o_2 the corresponding point over the right support o. Let g_2 be the intersection of the vertical at g with the load line, and e_3 the point where the vertical at e intersects a thread stretched from a_2 to o_2. Next stretch the thread from g_2 through the top of the step in the load line above f and mark the point where it crosses the vertical at e; mark a second point after passing the thread similarly through the bottom of the step. If e_3 lies between the last two points the criterion is satisfied.

To construct the influence diagram for a counter diagonal *FG*; or for *eF* when the counter *FG* is acting, and the main diagonal *EF* is not acting; or in *Gg* when either *FG* or *GH*, or both, are acting; it is desirable to re-draw a portion of the truss diagram in accordance with the proper conditions, and then construct the displacement diagram to correspond with it. To attempt to do so without re-drawing the truss diagram involves too large a risk of errors in construction.

If the Baltimore truss in Fig. 146 had sub-diagonal ties instead of struts it would change the form of several influence diagrams. The effect of this change is illustrated in the next article which gives the stress influence diagrams for a Pennsylvania truss.

Prob. 86. Draw the displacement diagram and influence line for the stress in the suspender *Cc*.

ART. 75. INFLUENCE LINES FOR PENNSYLVANIA TRUSS.

The influence diagrams shown in Fig. 147 are constructed for most of the members in one panel of a Pennsylvania truss corresponding to those of a Baltimore truss as illustrated in the preceding article. In this case, however, the sub-diagonals act as ties. The differences in the forms of the displacement diagrams are due both to the curved upper chord, and the kind of stress for which the sub-diagonals are designed. The influence line for the lower chord *eg* has the simple triangular form which is the same as if the truss panels were not sub-divided. That for the upper chord member *EG* forms a combination of the two triangles $a_1g_1o_1$ and $e_1f_1g_1$, since the secondary truss carries the panel load at f to the panel points F and G.

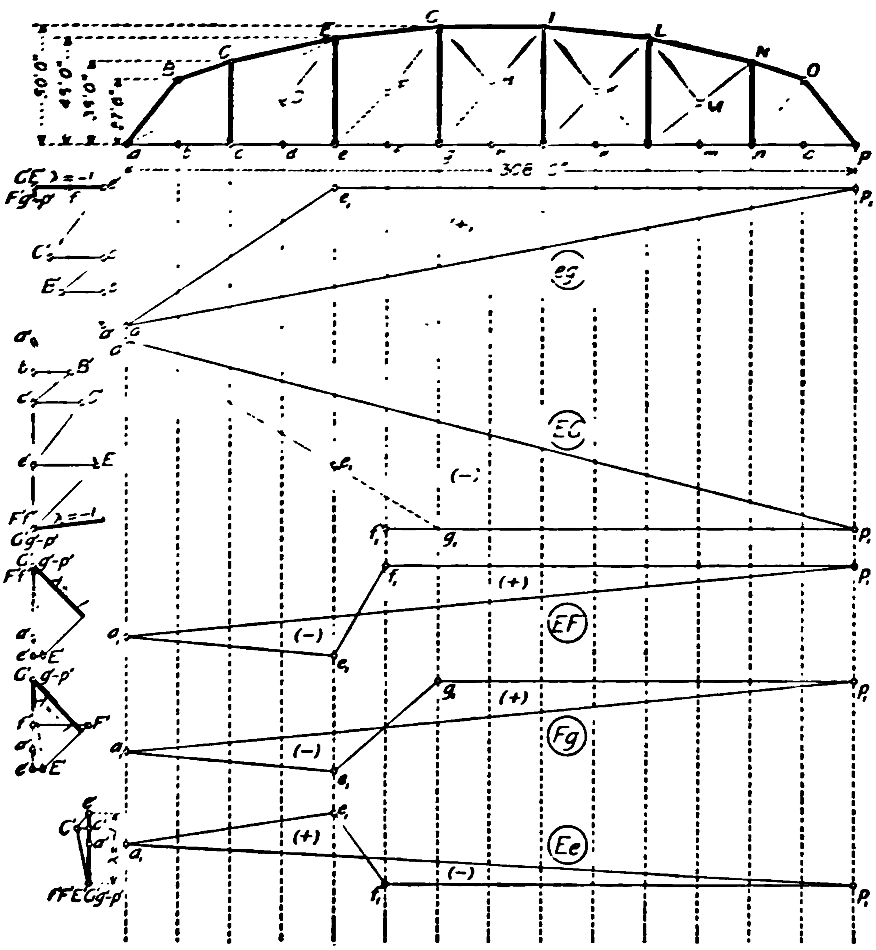

Fig. 147.

The criterion for loading for the greatest live load stress in *EG* is indicated by the following formula:

Load from a to $e+2$ times load from e to $f=Wl'/l$.

Expressed in general terms the left-hand member of the equation becomes: The load in the panels on the left of the section plus twice the loads in the panel cut by the section. In this case the vertical section is passed between *e* and *f* and the center of moments is at *g*. To satisfy the criterion an axle load must be placed at *f*. Using a similar notation to that employed in the preceding article the graphic method of applying this criterion is as follows: Let a_2, e_2, and p_2 be the respective points of intersection with the stepped load line of the verticals from *a*, *e*, and *p*. Let g_3 be the point where the vertical at *g* intersects a thread stretched from a_2 to p_2. Next stretch the thread from e_2 through the top of the load line step above *f* and mark its intersection with the vertical at *g*; mark a second point after changing the thread from e_2 through the bottom of the step. If g_3 falls between the last two points the criterion is satisfied.

In finding the stress in *GI* located in the middle panel of the truss it is impossible in advance to tell whether the counter diagonal *Hg* or *Hi* is acting. Assuming *Hi* to act the load is placed in position to satisfy the criterion given in the preceding paragraph. If it then be found that *Hg* acts for this position of the loads, the loading must be tested by the criterion given in the preceding article (for *eg* of the Baltimore truss in which the section is also passed through the panel *fg*), and another determination made as to whether *Hg* still acts or not.

Referring to the influence lines for the diagonals *EF*, *Fg*, and the vertical *Ee*, it will be observed that the horizontal projection of the middle influence line occupies one, two, and one panels respectively, while for the Baltimore truss in which sub-struts were used, it occupied two, one, and two panels respectively, or just the reverse arrangement.

Prob. 87. Construct the displacement diagram and influence line for *Ee* when the counter *Fe* on its right is acting, and check the three larger ordinates by computation.

ART. 76. INFLUENCE LINES FOR THE *K* TRUSS.

The skeleton diagram of the *K* truss in Fig. 148 represents the trusses in the bridge of the Atchison, Topeka, and Santa Fe Railway over the Arkansas River at Pueblo, Colo. It is the first simple truss span constructed in this country having the *K* form of web system, and was erected in 1915. A brief description of it was published in Engineering News, vol. 76, page 104, July 20, 1916. A half-tone illustration is given in Art. 90.

The displacement diagrams are constructed in the same manner as those in the preceding articles of this chapter. The direction of the vertical at the right end of the panel which contains the member and one of the extremities of that vertical are assumed to be fixed. For any of the vertical members it is necessary to extend the usual construction from panel point to panel point for a full panel to the left of the member before the end panel point can be located directly by means of an imaginary diagonal. For example, in the displacement diagram for F_2f, the position of g and the direction of gG are assumed as fixed; then the points g', G', F'_2, F', f', E'_2, e', and E' are located successively; and finally a' is located by inserting an imaginary diagonal Ea, $E'a'$ being perpendicular to Ea, and $e'a'$ perpendicular to ea.

The influence lines for ef and EF show the characteristic triangular form for all of the chord members, and in this respect are like those of the Parker truss in Fig. 145. The diagrams show that the centers of moments for both chords in the same panel lie in the same vertical. Since the upper chord member FG is horizontal, the magnitude of its stress for any loading is the same as that of the lower chord member fg in the same panel.

The influence lines for C_2d and C_2D give the typical form for all of the diagonals except those in the first two panels, which have influence lines like the corresponding diagonals in the Baltimore truss. The influence diagrams for C_2d and C_2D have exactly the same ordinates, since the inclinations of these

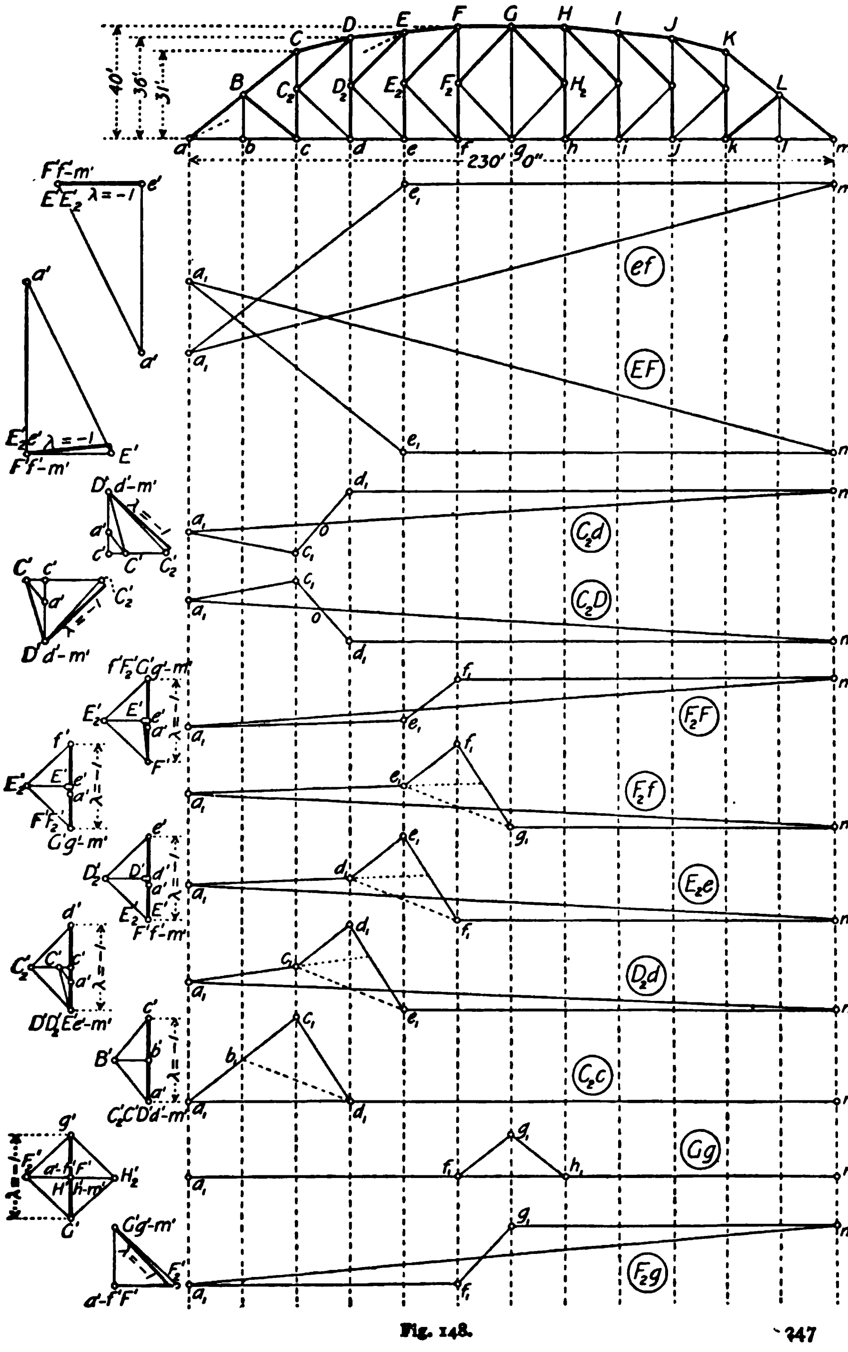

Fig. 148.

members are equal. Their stresses, however, are opposite in character. Their points of division *o* may be checked in the same manner as those of the diagonals in the Parker truss. The truss was designed so that the two diagonals in each panel have the same slope. Since the horizontal components of the diagonals in each panel must be equal, it is preferable to incline them equally and thus make the magnitudes of their stresses equal for any kind of loading.

The influence diagrams for D_2D and E_2E are similar in form to that shown for F_2F, the upper part of a post. It will be observed that its point of division is in the panel on its left instead of on the right as in the Parker truss. This is due to the fact that both of the diagonals adjacent to F_2F slope downward to the left.

The most interesting influence diagrams for this truss are those for the lower parts of the verticals and hence all of them are given in Fig. 148, viz.: C_2c, D_2d, E_2e, and F_2f. In all of these except C_2c, which is directly below the hip C of the truss, there is a break in the influence line on the left of the corresponding member similar to one of the lines for *eg* in Fig. 146, and for *EG* in Fig. 147. The dotted lines sloping downward to the right are drawn to show an interesting fact that the middle ordinate in the triangles $b_1c_1d_1$, $c_1d_1e_1$, $d_1e_1f_1$ and $e_1f_1g_1$ are all equal and measure 0.75 unit. This indicates that the lower verticals are influenced by a subsidiary truss action, like the chords in Figs. 146 and 147 referred to above. An examination of the displacement diagrams shows that the elevation of the left vertex of each triangle is midway between those of the other two vertices. This is due to the fact that both diagonals in the same panel have the same slope. The points of division for D_2d, E_2e, and F_2f are located about the same distance from one end in the corresponding panels. It will be noticed that these points of division occur in the panel on the right of the respective members, since the adjacent diagonals slope downward to the right. A criterion for loading for the greatest tension in these members could be readily deduced, but since the maximum ordinate is so close to the point of division it will be unnecessary. As this distance is about 14.4 feet, axle 3 of the COOPER loading should be placed at the panel point indicated by the maximum ordinate, the live load coming on from the left, thus bringing the pilot axle about 1.4 feet from the point of division. It will also be noticed that the hanger C_2c receives no stress from any load transmitted to the truss at all panel points on the right of d.

The influence diagram for Gg shows that its stress is due only to loads in the adjacent panel on each side of it. The maximum ordinate measures 0.5 unit. This indicates that the panel load at g is equally divided among the four diagonals in the two adjacent panels. The ordinate at g_1 in the influence line for F_2g measures a value equal to 0.25 unit multiplied by the secant of the angle which F_2g makes with the vertical. This angle is 45 degrees.

It may be added that the verticals in a K truss, while having to resist some compression, are primarily tension members. The upper diagonal on the left of a vertical corresponds to the compression diagonal of a Howe truss, while the lower diagonal corresponds to the tension diagonal of a Pratt truss; hence the vertical combines the functions of verticals in the web systems of both Howe and Pratt trusses.

Prob. 88. Construct displacement diagrams and influence lines for the stresses in C_2C and BC of the truss in Fig. 148.

CHAPTER IX.

DEFLECTION INFLUENCE LINES.

ART. 77. DEFLECTIONS OF BEAMS.

The methods of graphic statics are well adapted to obtain the deflection of any beam or girder, with cross-sections having either constant or variable moments of inertia, and with any kind of loading, whether concentrated, distributed, or both combined.

The fundamental equation in mechanics for the vertical deflection of a horizontal beam at any given point (see Mechanics of Materials, Art. 124) is

$$f = \int MM' \delta x / EI, \qquad (1)$$

in which M is the bending moment in any section of the beam due to the given loads, M' the bending moment due to an assumed load unity placed at the given point, δx a differential horizontal distance, E the modulus of elasticity, and I the moment of inertia of any cross-section.

Sometimes it is required to find the deflection of a beam at different points due to any given loading in a fixed position. In this case the bending moment diagram is first drawn and by treating it in turn as a loading diagram and drawing another equilibrium polygon to conform to certain conditions, this becomes a deflection diagram. The methods for doing this will be described and illustrated in subsequent articles.

At other times it is required to find the deflections at one point of a beam under different sets of loads, or under a set of moving loads. In this case it is necessary to construct a deflection influence line for the given point. The theoretical relations upon which the construction of deflection influence lines depend will now be presented.

Fig. 149*a* shows a cantilever beam with a load P at b and an assumed load of unity at a. The deflection of the beam at a, due to the load P only, can be found by means of equation (1) of this article. For the sake of distinction let subscripts be added to M and M' caused by P and unity respectively, thus: M_1 and M'_1. Now let the loads P and unity exchange positions as in Fig. 149*b*, the corresponding bending moments being designated M_2 and M'_2. Since the bending moment in any section due to a load in either position is proportional to the magnitude of the load, $M_2 = PM'_1$, and $M'_2 = M_1/P$; hence also $M_2M'_2 = M_1M'_1$. For the same beam equation (1) shows that the only variable is the quantity $\int MM'$. Therefore the deflection at a due to P located at b is exactly the same as the deflection at b due to P located at a. The same relation holds wherever b is located on the span.

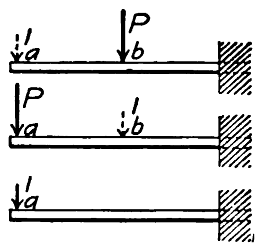

Fig. 149*a*.
Fig. 149*b*.
Fig. 149*c*.

Furthermore, if a graphic method is adopted the deflections of all points in the span are obtained by the same diagram whose construction is required to find that of any one point only. If the load at the end is made unity as in Fig. 149*c*, and the deflection diagram constructed by one of the methods given in the following articles, the diagram becomes a deflection influence line for a point at the end of the beam. To obtain the deflection at that point due to a concentrated load in any position, it is only necessary to measure the deflection ordinate at that position and to multiply its value by the magnitude of the given load.

The methods referred to require especial care with respect to the units in which different terms are to be expressed. The unit in which the deflection f is given by equation (1) when the unit for all the other terms are known may be found conveniently by cancellation. Let M be expressed in pound-inches, M' in inches since the assumed load is an abstract unity, E in pounds per square inch or pounds divided by square inches, and I in inches^4. The following equation may then be written:

$$\frac{\text{lb.-in.}\times\text{in.}\times\text{in.}}{(\text{lb./in.}^2)\text{in.}^4}=\frac{\text{lb.-in.}\times\text{in.}\times\text{in.}\times\text{in.}^2}{\text{lb.}\times\text{in.}^4}=\text{in.}$$

If one of the distances involved were expressed in feet it would change the result to feet. The same method may be employed in other combinations of terms to see whether all of the terms have been expressed in the proper units to give the result required.

It is to be remembered that equation (1) in this article gives the deflection due only to the bending moments in a beam and hence is in some degree approximate, although usually it fully satisfies the requirements of practice. If very precise values of deflections are required in any case, it is necessary to determine also the deflection due to shear. (See Mechanics of Materials, Art. 125.)

ART. 78. DEFLECTION OF A CANTILEVER GIRDER.

For example, let it be required to find the deflection at the end of a railroad turntable due to COOPER'S E50 loading (see Art. 39). When a locomotive is balanced on the turntable, preparatory to turning it as illustrated in Fig. 161 in Art. 90, each one of the pair of plate girders becomes a double cantilever, and each half of a girder is a cantilever with its neutral axis fixed in a horizontal direction over the middle support. Fig. 150*a* gives an elevation of one of these cantilevers on which the positions of the tender axle loads are indicated. The stiffeners of the plate girder are omitted on this diagrammatic representation. Fig. 150*b* is a diagram in which the ordinates represent the corresponding values of I, the moment of inertia of the cross-sections of the girder. The values of I are computed from the cross-section areas, and it is assumed that I does not change

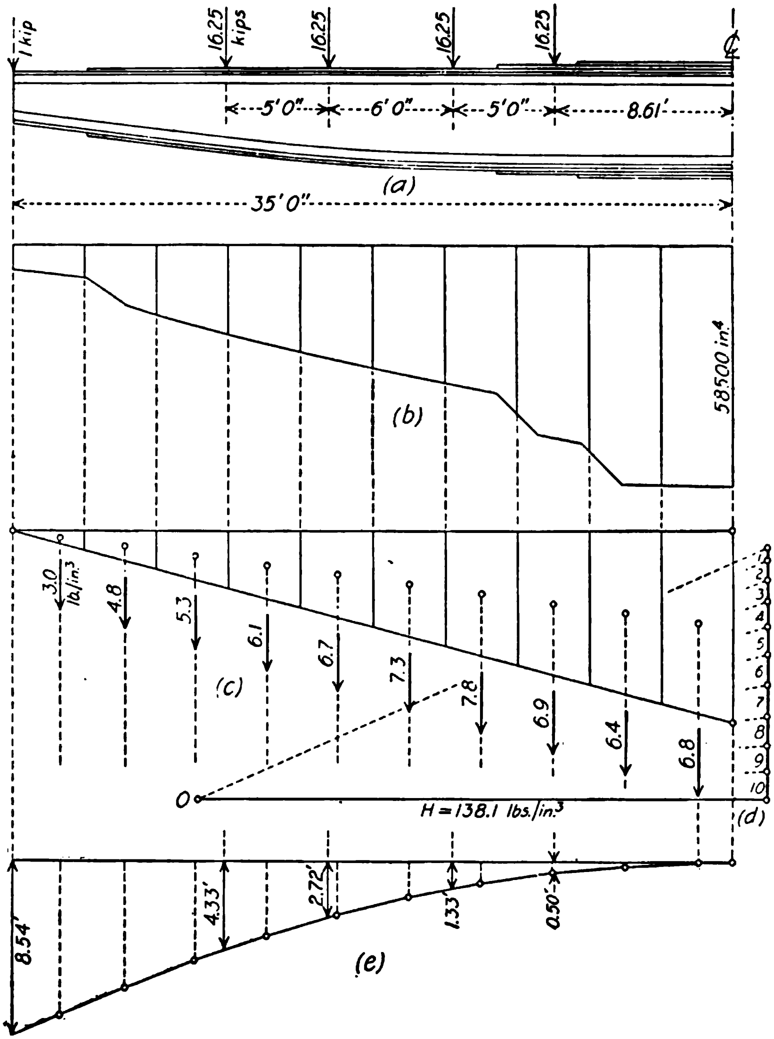

Fig. 150.

abruptly at the end of a cover plate but increases gradually to its full value in a distance of about 2 feet.

In a cantilever in which the horizontal distances x are measured from the free end $M' = -x$ in equation (1) of the preceding article, and if the span of the beam be divided into lengths Δx so as to apply graphic methods, the deflection may be determined with sufficient precision by equation (1) after making these substitutions, giving $f = \Sigma Mx \cdot \Delta x/EI$. This may be changed into the form

$$f = \frac{\Delta x}{E} \Sigma \frac{M}{I} \cdot x. \quad (2)$$

This summation can be most conveniently made by constructing an equilibrium polygon or bending moment diagram by treating the values of M/I as loads. In order to draw the deflection influence line for this cantilever girder it is necessary to place a load of 1 kip at its free end, since it is desired to find the deflection at that point, and then to construct the deflection polygon for that load, the girder being assumed as fixed at the support.

Fig. 150c shows the bending moment diagram for this load of 1 kip or 1000 pounds at the end of the girder. The span of the girder is divided into ten equal divisions of 3.5 feet = 42 inches each. The diagrams for I and M are both divided by vertical lines spaced the same distance apart. The average values of M, I and M/I for each division are given in the following table:

Division No.	M Pound-inches	I Inches^4	M/I Pounds/inches^3
1	21 000	7 000	3.0
2	63 000	13 200	4.8
3	105 000	19 800	5.3
4	147 000	24 200	6.1
5	189 000	28 000	6.7
6	231 000	31 600	7.3
7	273 000	34 800	7.8
8	315 000	45 600	6.9
9	357 000	56 000	6.4
10	398 000	58 500	6.8

Since $M \cdot \Delta x$ represents the area of one division of the moment diagram the loads M/I are to be applied at the centers of gravity of the division areas. If desired the loads may be taken as $M \cdot \Delta x/I$ instead of M/I, but this involves ten times as many multiplications by Δx.

The next step is to find the proper value for the pole distance H, remembering that the fundamental theory of the equilibrium polygon requires H to be laid off with the same scale as the loads, and hence H must be expressed in the same units. The same theory also gives $\Sigma(Mx/I) = Hz$, in which z is the ordinate of the equilibrium polygon (which in this case becomes a deflection polygon) at the limit to which the summation is made. Since the value of any deflection f is very small it is desirable that the ordinates z shall give the deflection as magnified n times, so as to be measured with greater precision; that is, $z = nf$. Making these substitutions in equation (2) there is obtained $f = (\Delta x/E)Hnf$ whence

$$H = E/(\Delta x \cdot n).$$

The value of E is 29 000 000 pounds per square inch, and $x = 42$ inches; hence the numerical value of the pole distance is

$$H = 29\,000\,000/(42n) = 690\,476/n$$

expressed in pounds divided by inches^3, which agrees with the units in which the values of M/I are expressed in the table. As H has to be laid off with the same scale as the load line which has a total value of 61.1 lb./in.^{3} it is found that a convenient value for n is 5 000, thus making $H = 138.1$ lb./in.^{3}. In Fig. 150*d* the loads are laid off in regular order and the pole is taken opposite the extremity of load 10 in order that the axis of the deflection polygon may be horizontal. The equilibrium polygon Fig. 150*e* is drawn in the usual manner, remembering that any side which lies between the lines of action of two forces must be parallel to the ray whose extremity lies between the same two forces in the force polygon.

Since the ordinate of an equilibrium polygon must be measured by the same scale of distances which is used to locate the positions of the forces on the beam, the deflection ordinate z at the left end is found to be 8.54 feet = 102.48 inches, and hence the deflection $f = 102.48/5\,000 = 0.0205$ inch, for a load of 1 kip applied at the end. As the deflection diagram is also a deflection influence line, the ordinates under each axle load are measured by the linear scale and found to be 4.33, 2.72, 1.33, and 0.50 feet respectively, their sum being 8.88 feet. The end deflection therefore for these four axle loads of 16.25 kips each is $8.88 \times 12 \times 16.25/5\,000 = 0.3463$ inch. This value is practically the same as 0.3459 inch, which was obtained by analytic computation, the lengths of division being 2 feet except at each end of the span, where they were a little longer.

If it be desired to make the divisions of the span unequal in order to conform somewhat to the variations of the moment of inertia, then the areas of the bending moment divisions must be obtained and the loads to be used for the deflection diagram are $M \cdot \Delta x/I$. The deflection influence line is really a curve which is tangent to the deflection polygon at the points of division. However, where the divisions are as short as in this example, it is not necessary to draw the curve, since it coincides so closely with the polygon.

Prob. 89. Determine the deflection of the extremity of the other half of the turntable girder referred to in this article, due to the axle loads which it supports.

ART. 79. ALTERNATIVE METHOD.

In Fig. 151*b* the diagram showing the variation of the moment of inertia is reproduced from Fig. 150*b* and a series of similar isosceles triangles are drawn so as to divide the span of the cantilever girder into lengths Δx which have a constant ratio to their respective average values of I. The relation follows from the constant ratio of the base to the altitude of each triangle. Hence the formula in the preceding article may be changed to

$$f=(\Delta x/EI)\Sigma Mx.$$

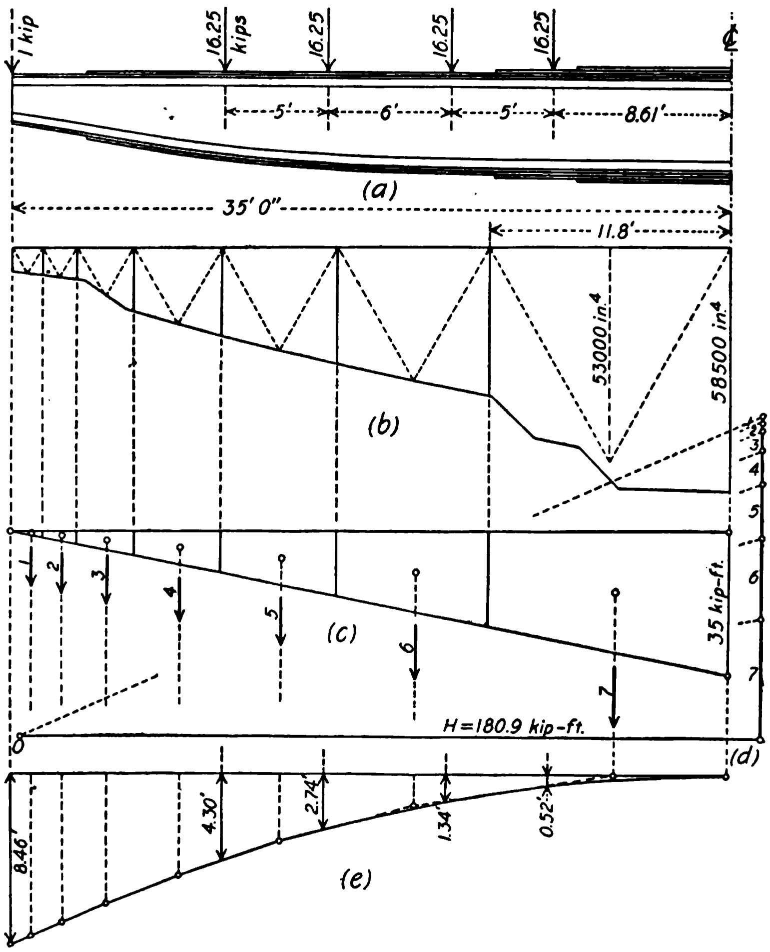

Fig 151.

The summation of Mx is again made most conveniently by constructing an equilibrium polygon or moment diagram in which the bending moment M is treated as a load.

The bending moment diagram due to the assumed load of 1 kip at the end is shown in Fig. 151*c*. Each value of M is the average value for its division and when treated as a load for the construction of another equilibrium polygon is applied at the center of gravity of its corresponding area. These middle ordinates may be transferred with the dividers to the load line without measuring their values by scale.

Before constructing this equilibrium polygon and its tangent curve which represents the deflection curve of the beam, it is necessary to determine the proper value of the pole distance H in Fig. 151*d*. Since $\Sigma Mx = Hz$ in which z is any ordinate at the limit for which the summation is made, and $z = nf$, as in the preceding article, there is obtained by substitution in the preceding equation $f = (\Delta x/EI)Hz = (\Delta x/EI)Hnf$, and hence

$$H = EI/(n \cdot \Delta x).$$

Since $E = 29\,000$ kips per square inch, I in the largest division is 53 000 inches[4], and the length of that division is $\Delta x = 11.8$ feet $= 141.6$ inches,

$$H = \frac{29\,000 \times 53\,000}{n \cdot 141.6} = \frac{10\,855\,000}{n} \text{ kip-in.} = \frac{904\,600}{n} \text{ kip-ft.}$$

As H has to be laid off with the same scale as the load line, it is found that a convenient value for n is 5 000, making $H = 180.9$ kip feet. With this value of H, the force polygon Fig. 151*d* is drawn, and the equilibrium polygon Fig. 151*e* constructed. The end ordinate z measured by the linear scale used in laying off the length of the beam is 8.46 feet = 101.5 inches. The deflection is therefore $101.5/5\,000 = 0.0203$ inch. The ordinates below the axle loads measure 4.30, 2.74, 1.34 and 0.52 feet, or a sum of 8.90 feet = 106.8 inches; hence the deflection at the end due to these loads is $106.8 \times 16.25/5\,000 = 0.3471$ inch.

As the deflection polygon in this case has such long sides it is necessary to draw a smooth curve tangent to the sides respectively at the points of division, before the ordinates under the axle loads are measured. Only a part of the curve is shown in Fig. 151*e* on account of the reduced size.

In Art. 78 the bending moments M were divided by the respective moments of inertia I, before being laid off on the load line. In this article the span was divided into unequal divisions Δx so as to make the ratio of $\Delta x/I$ constant. There remains a third method of procedure in which the pole distance H is changed in direct proportion to the average value of I for each division.

In case it is desired to find the deflections for one or more loads, which are fixed in position at a number of different points on the span, then the simplest procedure is to draw first the bending moment diagram for the given loading, and afterward to construct the deflection diagram by either of the three methods referred to in the preceding paragraph. The magnitude of any ordinate will give the total deflection at the location of that ordinate. By using the bending moment diagram for the four axle loads in the example considered in this article, and adopting the third method of drawing the deflection polygon, the deflection at the end of the girder was found to be 0.343 inch, and under the axle loads 0.210, 0.146, 0.078, and 0.032 inch respectively. In this case the moment diagram was divided into only 5 divisions of unequal length.

Usually the method described and illustrated in Art. 78 is the most advantageous, since the average values of I are more readily obtained than in the other methods. Where the values of I change more or less abruptly it requires a number of trials to divide the span so as to make $\Delta x/I$ constant, and in which I is really the average value for its division. Under some conditions, however, this method may be preferable.

When the moment of inertia is constant the construction is simplified still further, since I is eliminated from the summation.

For some cases of special loading the analytic method may then be more advantageous than the graphic method.

Prob. 90. Check the value of the deflection obtained in Prob. 89, by the method explained in this article.

ART. 80. DEFLECTION OF A SIMPLE BEAM.

The same reciprocal relation which was shown in Art. 77 to exist between the deflections and loads at any two points on a cantilever beam apply likewise to any other type of beam. Hence the simplest graphic determination of the deflection of a simple beam at any given point under any loading consists in the construction of a deflection polygon for a unit load placed at the given point. This polygon is also the deflection influence line by means of which the deflection may be readily obtained for any other loading.

In Fig. 152 a simple beam is shown with a load of 1 kip applied at six-tenths of its span from the left end. The reactions are therefore 0.4 and 0.6 kip respectively. Let the tangent of the neutral axis of the beam at a point under the load be assumed as fixed in a horizontal direction. The two parts of the beam may then be regarded as cantilever beams with upward loads of 0.4 and 0.6 kip applied at their respective ends. The deflection polygons for both cantilevers are drawn according to the method given in Art. 78. Since the bending moments are positive the deflections of the ends are upward from the fixed tangent or axis, as indicated in the lower diagram of Fig. 152. But according to the actual condition of the beam the supports are fixed in elevation, making the deflection of the beam at those points equal to zero; hence the required vertical deflection at any point must be measured from the closing line a_1b_1 as an axis.

The entire deflection polygon with its closing line is found to be identical with an equilibrium polygon constructed by treating the entire bending moment diagram for the simple beam as a load diagram. It was assumed that the tangent to the neutral axis of the beam under the load was fixed in a horizontal direction. The assumed direction is really immaterial, since the magnitudes of vertical ordinates in an equilibrium polygon are independent of the position of the pole in the force polygon, provided the pole

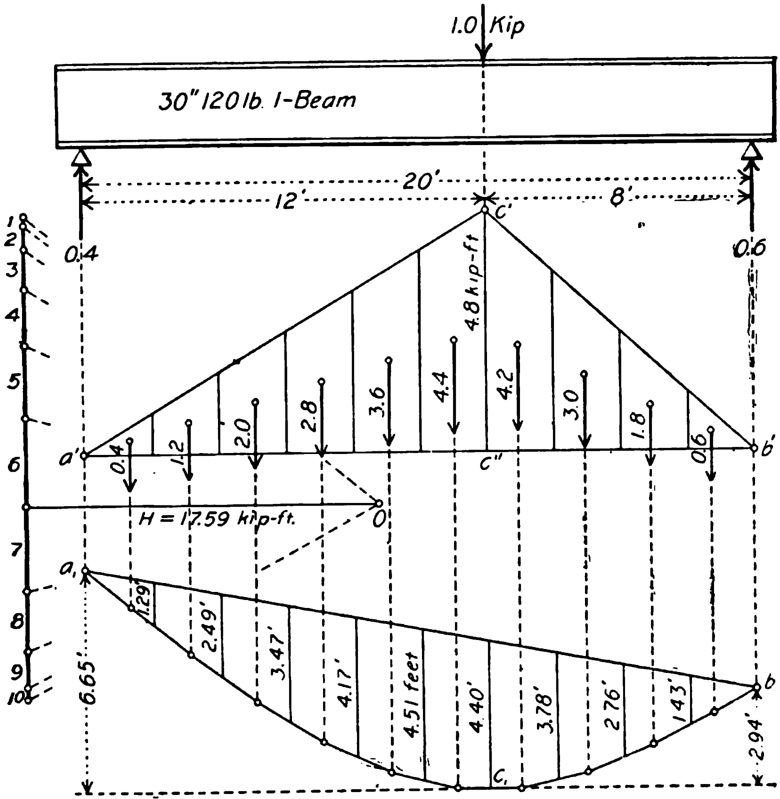

Fig. 152.

distance H remains the same. (See Art. 7.) If the axis a_1b_1 be made horizontal and the same end ordinates laid off as before the true inclination of the tangent at c_1 will be obtained.

Let the beam represented in Fig. 152 be a 30-inch steel I beam weighing 120 pounds per linear foot, and having a moment of inertia of cross-section of 5239.6 inches^4. It is required to find its deflection for a span of 20 feet, under a uniform load of 9300 pounds per linear foot, at a section 12 feet distant from the left support. The load includes the weight of the beam itself.

The span is divided into ten equal parts, making each division 2 feet long. A load of 1 kip is placed at the point whose deflection is to be determined and the bending moment diagram constructed. The maximum ordinate $c'c''$ equals $1 \times 12 \times 8/20 = 4.8$ kip-feet. The value of the middle ordinate of each division of the bending moment diagram is given on the drawing expressed in kip-feet. The average or middle ordinate in each division is to be treated as a load in constructing the deflection polygon. The sum of those on the left of c'' is 14.4 kip-feet, and of those on the right, 9.6 kip-feet.

In order to make the direction of the deflection polygon horizontal at c_1 or under the load of 1 kip, the pole is located in a horizontal ray through the extremity of load 6 on the load line. Since $H = EI/(n \cdot \Delta x)$, its numerical magnitude is

$$H = \frac{29\,000 \times 5239.6}{24n} = \frac{6\,331\,000}{n} \text{ kip-in.} = \frac{527\,600}{n} \text{ kip-ft.}$$

In order to secure a deflection polygon of good proportions it is found convenient to take $n = 30\,000$, making $H = 17.59$ kip-feet. Upon constructing the deflection polygon and measuring the ordinates at a_1 and b_1 by the linear scale it is found that the left end of the beam deflects upward $6.65/n$ feet and the right end $2.94/n$ feet from the tangent to the neutral axis at c. Upon drawing the closing line a_1b_1 and measuring the ordinates at the ends of the divisions, their values are found to be 0, 1.29, 2.49, 3.47, 4.17, 4.51, 4.40, 3.78, 2.76, 1.43 and 0 feet respectively.

Since the load is uniformly distributed it is necessary to find the area of the deflection influence diagram or the sum of the average ordinates in divisions of one foot each. By means of Simpson's rule the area is found to be 57.04 square feet, or the sum of the ordinates is 57.04 feet for the one-foot divisions. The deflection for a load of 9.3 kips per linear foot is therefore

$$f = 57.04 \times 9.3/30\,000 = 0.01768 \text{ ft.} = 0.2122 \text{ in.}$$

By means of the equation of the elastic line for this case in which $x = 0.6l$ (see Mechanics of Materials, Art. 55) the deflection is computed to be 0.2098 inch, which differs from the value obtained graphically by a little over 1 per cent. This difference could have been reduced by using larger scales. The scales employed on the original diagram for Fig. 152 are 3 feet to 1 inch and 5 kips to 1 inch. The maximum deflection of this beam at the center of the span computed by means of the ordinary formula is 0.220 inch.

Prob. 91. A simple beam having a span of 20 feet consists of a 20-inch I-beam weighing 65 pounds per foot, with a moment of inertia of 1169.5 inches^4. The load varies uniformly from zero at one end of the span to 11 kips per foot at the other end. Find the deflection at intervals of 2 feet throughout the span by means of a deflection polygon.

ART. 81. DEFLECTION OF A CRANE GIRDER.

The simple beam in Fig. 153 represents one of a pair of box girders in a traveling crane similar to that illustrated in Fig. 162 of Art. 90. It has two web plates $\frac{1}{4}$ inch thick. The upper cover plate is $20 \times \frac{1}{2}$ inch, and the lower one $20 \times \frac{3}{8}$ inch. The two upper flange angles are $5 \times 3 \times \frac{1}{2}$ inch and the lower ones $5 \times 3 \times \frac{3}{8}$ inch, their longer legs being vertical. The short flange angles at each end are $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$ inch, and $48\frac{3}{4}$ inches long. Some of the dimensions of the girder are given on the diagram. Stiffeners and minor details are omitted. The lower flange has a parabolic curve. The depth back to back of flange angles is $7\frac{7}{8}$ inches above the supporting girder, $26\frac{1}{2}$ inches at the end of the curved bottom flange, and $48\frac{1}{2}$ inches at the center of the span. The moments of inertia at these sections are 250, 5760, and 21 630 inches^4. The distance between the second and third of these sections was divided into five equal parts, and the moments of inertia computed at the ends of divisions, giving the values 9970, 14 460, 18 340, and 20 760 inches[4]. The deflec-

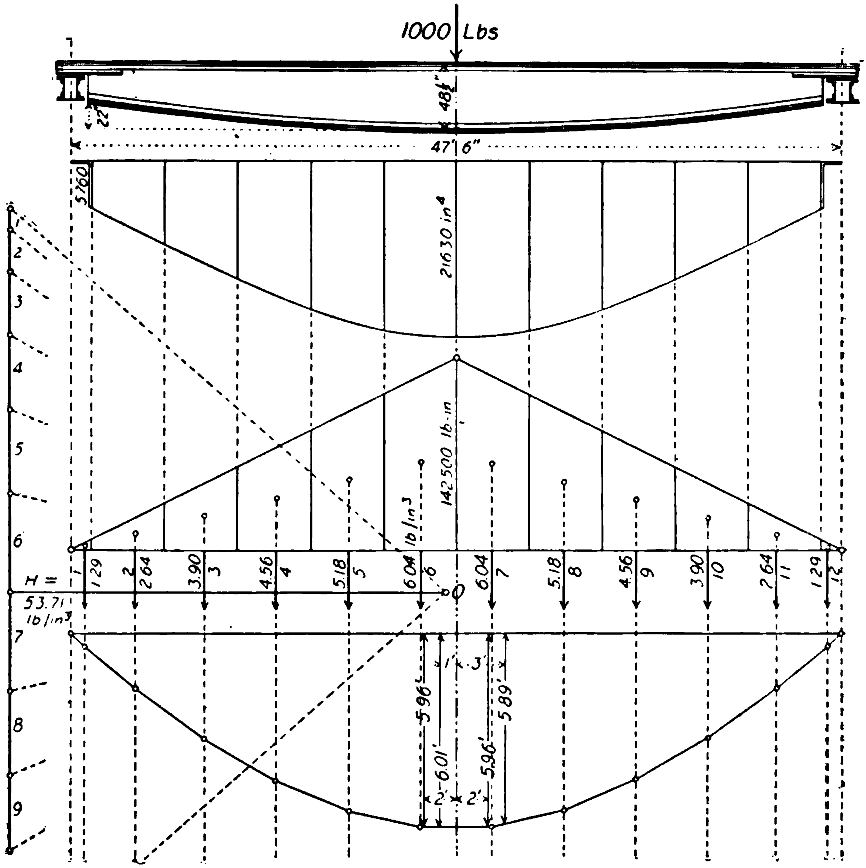

Fig. 153.

tion of the girder at the center of the span is to be found due to a load of 30 kips on each girder.

On account of the abrupt change of section next to the supporting girders the span is not divided into equal parts throughout.

All divisions except one at each end are 4.5 feet long, the end division being 1.25 feet in length. The values of the middle ordinates in the divisions of the bending moment diagram due to a load of 1 000 pounds at the center of the span are given in the following table. The table also contains the average values of the moments of inertia for the respective divisions and the corresponding values of M/I.

M Pound-inches	I Inches^4	M/I Pounds/inches^3	Loads
3 750	810	4.63*	1,12
21 000	7 960	2.64	2,11
48 000	12 320	3.90	3,10
75 000	16 460	4.56	4,9
102 000	19 700	5.18	5,8
129 000	21 360	6.04	6,7

Since the end divisions are only 15 inches long instead of 54 inches, the first $M/I = 4.63$ lbs./in.^{3} must be multiplied by 15/54, giving 1.29 lbs./in.^{3}, before it is laid off on the load line in constructing the deflection influence line. The magnitude of the pole distance is

$$H = \frac{29\,000\,000}{54n} = \frac{537\,060}{n} \text{ lbs./in.}^3.$$

Since the total load line measures 47.22 units, a convenient value to assume for n is 10 000, making $H = 53.71$ units. The deflection polygon is constructed in the same manner as described previously. The electric hoist and its live load of 60 000 pounds are carried by a four-wheeled trolley which is supported by the rails on top of the pair of crane girders. The two axles of the trolley are spaced 4 feet, hence it is necessary to measure two ordinates with this spacing on the deflection influence line. If both ordinates are 2 feet from the center of the span they measure 5.96 feet, while if one is 1 foot from the center and the other 3 feet on the other side of the center they measure 6.01

* $4.63 \times 15/54 = 1.29$; reduced to an equivalent for division of 54 inches.

and 5.89 feet. The larger sum is 11.92 feet, and therefore the deflection at the center due to a load of 15 000 pounds on each trolley wheel is

$$f = 11.92 \times 12 \times 15/10\,000 = 0.215 \text{ inch.}$$

In this formula the load is introduced as 15, since 15 000 is 15 times as large as the load of 1 000 pounds which was placed at the point whose deflection was to be found.

To find the total deflection of the crane girder it is necessary to consider the weight of the trolley and the electric hoist as well as the distributed weight of the girder itself, the squaring shaft, the bridge walk and brackets which it carries.

Prob. 92. Find the greatest deflection of the above girder at the quarter point of the span due to the same load.

ART. 82. DEFLECTION INFLUENCE LINES FOR TRUSSES.

In Chap. VII a method was given for finding the deflection of the different panel points of a truss under a load which is fixed in position. This method is used in determining the elevation of the blocking required for the erection of bridge trusses, in which case the change of length for each truss member is computed for the sum of its stresses due to both dead load and full live load. The position of the live load is that which causes the greatest bending moment at the center of the span. It is also used in finding the deflection of different panel points due to a given live load when a truss bridge is being tested by observing the actual deflections under that live load in a specified position, in order to compare the results with the theoretic values previously found.

If, on the other hand, it is desired to find the deflection of any given panel point under different loads, or under a moving live load, it is necessary to construct a deflection polygon for a load of 1 kip or 1 000 pounds placed at that point. This polygon is also a deflection influence line for the given panel point, on account of the reciprocal relation between loads and deflections for beams, as shown in Art. 77. The same relation between the bending moments M and M' indicated in that article exists likewise beween the stresses S and T, since the formula for the deflection of a truss involves the product ST (see Roofs and Bridges, Part I, Art. 87) just as the formula for the deflection of a beam includes the product MM'.

The most convenient method to find the stresses in the truss members due to a load of 1 kip at the given panel point is to construct an ordinary stress diagram unless the chords are both horizontal, in which case the analytic method furnishes the quickest solution. After the values of the changes in length λ due to these stresses are known, the displacement diagram is constructed and afterwards the deflection diagram, or deflection influence line, as described and illustrated in Arts. 69 and 72.

Prob. 93. Construct the deflection influence line for panel point c of the truss in Fig. 137, Art. 69.

CHAPTER X.

REFERENCE LITERATURE OF GRAPHICS.

ART. 83. LOADS AND STRESSES FOR ROOF TRUSSES.

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ART. 85. LIVE LOADS FOR RAILROAD BRIDGES.

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Fig. 154. Interior View of Power House of the Niagara Falls Power Co., Showing Fink Roof Trusses Supporting Purlins of King-post Trussed Beams. The roof sheathing is supported directly by the purlins.

Fig. 155. Interior View of a Freight Shed Showing Steel Roof Trusses with Wide Spacing and Riveted Truss Purlins.

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Fig. 156. Foot Bridge with Riveted Simple Truss Spans over West Street at the Liberty Street Station of the Central Railroad of New Jersey, New York City. Built in 1909. Span 101 feet, 2 inches.

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Fig. 157. Lehigh Valley Railroad Bridge over the Susquehanna River at Towanda, Pa. It contains thirteen double-track deck plate-girder spans 129.5 feet long, and one span 120 feet long. Completed in 1907.

Fig. 158. Lock Raven Highway Bridge No. 2 of the Baltimore City Water Department, Baltimore, Md. The Parker trusses have a span of 297 feet, and are supported near the tops of the steel towers. Erected in 1914.

Fig. 150. The Longest Simple Truss Span in the World. Span 720 feet. C. B. & Q. R.R. and N. C. & St. L. Ry. Bridge over the Ohio River at Metropolis, Ill. The Photograph was taken in 1917 especially for this illustration and before adjacent span on the left was erected.

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ART. 90. DEFLECTIONS AND INFLUENCE LINES.

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DEFLECTION OF BEAMS.—Deflection of Beams by Graphics. Willibald Trinks. Trans. Am. Soc. M.E., 1903, v. 24, p. 116,

Fig. 160. First Simple Truss Bridge with K System of Web Members Built in America. Atchison, Topeka and Santa Fe Railway Bridge over Arkansas River at Pueblo, Colo. Erected in 1915. Span 230 feet. The pier in the river supports the Denver & Rio Grande Railroad bridge shown directly behind the K-truss bridge. A city highway bridge is also shown in the background.

Fig. 161. Turntable of the Central Railroad of New Jersey at Dunellen, N. J. It is 70 feet long and designed for a locomotive weighing 187.5 tons.

Fig. 162. Electric Traveling Crane in the Hydraulic and Steam Laboratory of the Massachusetts Institute of Technology, 1916.

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APPENDIX.

ANSWERS TO PROBLEMS.

Prob. 1. 56.8 pounds, making an angle of 37° 35′ with the smaller force.

Prob. 2. 43.6 pounds, 36° 35′ and 83° 25′.

Prob. 4. 107.8 pounds and 129° 20′.

Prob. 5. 2 800 pounds.

Prob. 6. $S_2 = + 5.56$, $S_3 = - 3.27$, $S_4 = - 5.86$, and $S_5 = +$ 5.95 tons.

Prob. 8. $S_1 = - 43.6$, $S_2 = S_4 = + 109.1$, and $S_3 = + 43.6$ pounds.

Prob. 9. Resultant = 279.7 pounds, and angle with greater force = 1° 45′.

Prob. 11. Resultant = 4 tons, is parallel to forces and 6 feet from greater force.

Prob. 12. 177.1 and 192.9 pounds.

Prob. 13. Maximum shear = ± 4 000 pounds, maximum moment = + 20 000 pounds-feet.

Prob. 15. Maximum shear = − 6 tons, maximum moment = − 30 tons-feet.

Prob. 16. Maximum shear = − 6 tons, maximum moment = − 30 tons-feet.

Prob. 17. 0.70 inches from the back of channel iron.

Prob. 18. 500 pounds, 15.27 feet from the first force.

Prob. 19. $I = A \times A' = 7.10 \times 10.81 = 76.75$ inches^4, and $I' = A \times A'' = 7.10 \times 0.51 = 3.62$ inches^4.

Prob. 22. Stress in $AC = BC = -2\,290$ pounds, and in $CD = +2\,050$ pounds.

Prob. 23. Apex loads = 1.48, 1.20 tons; reactions = 4.44, 3.60 tons.

Prob. 24. 2.79, 8.37, 2.04, and 6.12 tons.

Prob. 25. Apex loads = 2.15 and 1.61 tons. Dead load stresses are: in upper chord, − 12.03, − 9.78, − 10.10; in lower chord, + 10.75, + 6.45; and in braces, − 2.30, − 2.30, + 4.29 tons. The corresponding snow load stresses are, − 9.00, − 7.32, − 7.56; + 8.05, + 4.83; − 1.72, − 1.72, + 3.21 tons.

Prob. 27. $AB = 2\,035$, $BC = 2\,750$, and $CD = 814$ pounds, the normal wind pressures being 38.2 and 15.6 pounds per square foot.

Prob. 28. Apex load = 1.86 tons; reactions = 3.84 and 1.74 tons. Stresses in upper chord, − 5.82, − 6.75, − 5.36, − 3.49, − 3.49, − 3.49; in lower chord, + 6.51, + 4.43, + 2.35, + 2.35, + 2.35; and in braces, − 2.08, + 2.94, − 3.12, + 3.75, 0, 0, 0, and 0 tons.

Prob. 32. Apex load = 2.05 tons; reactions at free end, 2.30 and 4.99 tons. Stresses for wind on fixed side are: in upper chord, − 8.93, − 7.45, − 5.98, − 4.51, − 5.04; in lower chord, + 11.22, + 8.98, + 6.73, + 4.48; and in braces, − 2.52, + 1.15, − 3.21, + 2.30, − 4.11, + 3.45, and 0 tons. Lower chord stresses are diminished 3.75 tons for wind on free side.

Prob. 34. Dead apex loads = 0.70, 1.40, 1.40, etc.; snow apex loads = 0.43, 1.08, 1.30, etc.; and wind apex loads = 1.74, 2.35, and 0.70 tons. Maximum stresses in upper chord, − 11.0, − 10.3, − 8.7, − 9.4; in lower chord, + 9.4, + 8.4, + 6.4; and in braces, + 1.9, + 2.7, + 1.7, + 3.3 tons. Minimum stresses, − 3.9, − 3.3, − 3.4, − 4.0; − 0.7, + 0.8, + 2.3; + 1.0, − 1.2, − 0.2, and − 0.5 tons.

Prob. 36. Dead, snow, and wind apex loads are 0.91, 0.84, and 1.49 tons. Maximum stresses in upper chord, − 14.45,

-11.38, -9.56; in lower chord, $+13.96$, $+9.41$; and in braces, -3.82, $+2.06$, -3.21, $+3.44$ tons. Minimum stresses, -5.09, -4.07, -3.39; $+4.55$, $+3.34$; -1.02, $+0.55$, -0.85, and $+1.21$ tons.

Prob. 37. $cd = ce = -4.31$, $cg = ch = -1.73$, $ad = +3.91$, $bf = +1.26$, $bh = +1.57$, $de = 0$, $ef = +3.55$, $fg = +0.39$, and $gh = 0$ tons.

Prob. 38. Apex loads = 0.6, 1.2, and 0.6 tons. Designating the members as in Fig. 51, the stress $bf = -4.15$, $cg = -3.61$, $dk = dl = -2.06$, $ef = +3.76$, $eh = +1.50$, $el = +1.88$, $fg = -1.08$, $gh = +2.36$, $hk = +0.47$, and $kl = 0$ tons.

Prob. 39. Dead panel load per truss = 1.98, live = 4.70 tons.

Prob. 40. Panel loads = 2.83 on upper, and 5.67 tons on lower chord. Stresses in upper chord, -39.4, -47.3, -47.3; in lower chord, $+23.6$, $+23.6$, $+39.4$, $+47.3$; and in web members, -34.7, $+5.7$, $+23.2$, -11.3, $+11.6$, 0, and 0 tons.

Prob. 41. Maximum stresses in upper chord, -46.6, -123.3, -169.2, -184.5; in lower chord, $+92.0$, $+153.3$, $+183.9$ tons. Minimum stresses, -12.2, -31.5, -43.0, -46.8; $+23.1$, $+38.5$, $+46.2$.

Prob. 43. Panel load = 0.79 tons. Stresses in the chords, 3.4, 6.2, 8.4, 9.9, 10.9, 11.2; in diagonals, -5.5, $+4.5$, $+3.5$, $+2.5$, $+1.5$, $+0.5$; and in verticals, $+0.8$, -2.8, -2.0, -1.2, -0.4, and 0 tons.

Prob. 44. Panel load due to truss is 1.074, and that due to train is 4.0 tons. Stresses in upper chord for south wind, 0, $+2.0$; for north wind, -2.0, -2.7 tons. In the lower chord, 0, $+15.9$, $+25.4$; -15.9, -25.4, -28.5 tons. Maximum wind stresses in diagonals of upper lateral system, $+2.6$, $+0.9$; in struts, -1.6, -1.1 tons. In lower lateral system, $+20.3$, $+13.3$, $+7.3$; -12.7, -8.3, and -5.1 tons.

Prob. 46. The greatest reduction of stress is in *RC*, and equals $4.7 \div 76.4 = 6.2$ per cent.

Prob. 47. Maximum stresses in upper chord, -56.0, -53.2,

— 52.1; in lower chord, + 50.0, + 50.0, + 51.3; in main ties, + 8.3, + 8.7; in counter ties, + 7.3, + 8.0; and in verticals, + 10.0, + 9.4, + 9.0 tons. Minimum stresses in chord, — 14.0, — 13.3, — 13.0, + 12.5, + 12.5, + 12.8; in diagonals, 0; and in verticals, + 2.5, + 0.2, — 0.3 tons.

Prob. 48. Maximum stresses in the chords, — 100.0, + 109.7, + 105.9, + 103.0, + 101.1, + 100.1; in main ties, + 9.0, + 10.2, + 10.9; in counters, + 7.7, + 9.0, + 10.2, and in verticals, — 10.0, — 12.4, — 14.2, — 15.2 tons. Minimum stresses in the chords, — 30.0, + 32.9, + 31.8, + 30.9, + 30.3, + 30.0; in diagonals, 0; and in verticals, — 3.0 tons.

Prob. 51. 42.5 tons.

Prob. 52. Dividing the span into eight equal parts, the flange stresses at the sections are 0.0, 16.0, 26.4, 32.6, and 35.1 tons. The absolute maximum is 35.2 tons at 4 inches from center of girder. The shears are 20.1, 16.3, 12.8, 9.4, and 6.1 tons.

Prob. 53. Maximum stresses in chords, 86.2, 138.1, 167.1; in end post, — 135.2; in main ties, + 95.4, + 60.1, + 28.0; in counter ties, + 3.5, + 28.0; and in verticals, + 40.8, — 48.3, and — 23.5 tons. Minimum stresses in the chords, 20.5, 34.2, 41.0; in end posts, — 32.2; in main ties, + 17.9, 0, 0; in counter ties, 0, 0; and in verticals, + 6.3, — 1.9, and — 1.9 tons.

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